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An optimization method for the distance between exits of buildings considering uncertainties based on arbitrary polynomial chaos expansion

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ABSTRACT

The distance between exits is an important design parameter in fire safety design of buildings. In order to find the optimal distance between exits under uncertainty with low computational costs, a surrogate model (i.e. approximation model) of evacuation time is constructed by the arbitrary polynomial chaos expansion. Through a two-stage nested Monte Carlo simulation of this surrogate model, the optimal distance between exits under uncertainty is found efficiently. In order to demonstrate the proposed method, a single room with two exits is presented as a fire compartment and uncertainties of occupant density and child-occupant load ratio are also considered. In this case, the results showed that the optimal distance between exits changes with the level of probability of evacuation time, and there is a critical level of probability for the transition of the optimal value of the distance between exits. Furthermore, the traditional Monte Carlo simulation method is used to compare the accuracy of the surrogate model with the computer evacuation model FDS+Evac developed by the VTT Technical Research Centre of Finland [1]. The results indicate that the proposed surrogate-based optimization method can achieve a similar accuracy with much lower

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computational costs.

Keywords**:** Evacuation time; Uncertainty analysis; Performance-based fire protection design; Optimization under uncertainty; Distance between exits.

1. Introduction

The distance between exits is a critical issue in fire protection design of buildings, which is generally determined within a certain range according to the current prescriptive-based fire protection codes of buildings. For example, the edge distance between adjacent exits should be larger than 5m in a fire compartment of public buildings according to the Chinese code GB50016-2014 [2]. However, for larger distance between exits, the travel distance of some occupants may be increased, which may reduce evacuation efficiency of crowds. Moreover, once available safe egress time (*ASET*) is larger than required safe egress time (*RSET*), buildings are regarded as safe enough without the consideration of the optimal value of the distance between exits in performance-based fire protection design of buildings [3]. Besides, due to the randomness of the fire and occupant characteristics [4], there may be a high degree of uncertainty in initial conditions and processes of occupant evacuation. Hence, in order to conduct on a cost-effective fire protection design of buildings, the distance between exits need to be optimized with consideration of uncertainties.

Great attention has been paid to uncertainties in evacuation modeling by many researchers. Sources of uncertainty in evacuation modeling are generally classified into 4 types: model input uncertainty; measure uncertainty; intrinsic uncertainty and behavioural uncertainty [5]. Based on a lattice-gas model, Song et al.[6] adopted the mean field theory to develop an evacuation model considering uncertainty in the number of pedestrians. Moreover, Averill et al.[7] employed a Modified-Markov Modeling approach to propose a grid-based evacuation model, which can be used to deal with uncertainty in the number of occupants. Additionally, Xie et al.[8] utilized the polynomial chaos expansion to solve uncertainties in input parameters with the purpose of reducing the computational cost of uncertainty analysis of evacuation time. Besides, in order to address the randomness of occupant characteristics, distributions of stochastic variables (e.g. pre-movement time, walking speed, body diameter) are embedded in more computer evacuation models, such as buildingEXODUS [9], Simulex [10,11], FDS+Evac [1] and Pathfinder [12]. Meanwhile, random numbers/seeds are used in evacuation modelling to deal with the space conflict, exit choice, queuing behavior, etc. [13]. With the aim of accurately evaluating the performance capability of passenger evacuation, Vassalos et al. [14] adopted the Monte Carlo simulation approach to obtain the cumulative probability distribution (CDF) of evacuation time under uncertainties regarding to human behavior. In order to address the impact of human behavior on evacuation model prediction, based on functional analysis Ronchi et al. [13] proposed a quantitative method to determine the suitable number of simulations for the same evacuation scenario.

Recently, the distance between exits has been widely studied by some researchers. Based on a two-dimensional cellular automaton model, Zhao et al.[15] investigated the effect of the distance between exits on evacuation time under deterministic evacuation scenarios, and suggested that the layout of exits should be symmetrical for a higher crowd evacuation efficiency and the optimal value of the distance between exits is approximately equal to 3/10 of the total length of the wall, which is independent of exit width. Moreover, in order to save the computational cost of fire protection design of buildings, Tavares et al. [16] proposed a surrogate model-based optimization method to find the optimal location of exits under deterministic evacuation scenarios, which couples numerical optimization techniques to a deterministic response surface method with the buildingEXODUS evacuation model. In addition, based on evacuation simulations using the computer model Simulex, Tavares et al.[17] showed that the efficiency of crowd evacuation is significantly affected by the relative distance between exits, which can be used to determine the suitable exit locations for evacuation design of buildings. Zhou et al.[18] adopted a cellular automation model to explore occupant evacuation dynamics, and indicated that the efficiency of crowd evacuation will be reduced by the improper distance between exits. Besides, Jiang [19] also investigated the impact of the configuration of exits on the efficiency of crowd evacuation and showed that the optimal value of the distance between exits is 1/5 of the length of the wall. Furthermore, Rao et al. [20] examined the importance of the distance between exits in occupant evacuation time based on a cellular automation model and indicated that the distance between exits can have a more significant impact on occupant evacuation than exit width in complex buildings.

As discussed above, it can be seen that previous research focused on the optimal design of the distance between exits under deterministic evacuation scenarios. However, occupant evacuation is extremely complex and highly uncertain, which is affected by many uncertain factors such as pre-movement time, occupant density, crowd type, etc. Thus, the optimal distance between exits under uncertain evacuation scenarios is an important and interesting topic.

2. Optimization method for the distance between exits under uncertainties

Occupant evacuation is extremely complex, which makes the treatment of uncertainties a very important aspect in evacuation modeling. To address evacuation modelling uncertainties, Tavares et al. [21] presented different solutions, e.g. Design of Experiments (DoE) techniques and functional analysis. For the optimization of the distance between exits under uncertainties, the most straightforward method is two-stage nested Monte Carlo simulation of computer evacuation models.

However, this method is highly time-consuming due to the complexity of occupant evacuation process. In order to save the computational cost of optimization under uncertainties, the most obvious approach is directly coupling the Monte Carlo simulation with numerical optimization techniques in computer evacuation models, but it still requires a numerous computer evacuation simulations. To significantly reduce the computational cost, a surrogate model-based optimization method is proposed to determine the optimal distance between exits under uncertainties. The arbitrary polynomial chaos expansion, which is one of stochastic response surface methods, can be used to construct the surrogate model of the output under uncertainty with fewer computer simulations [22]. In this paper, given uncertainties in evacuation scenarios (i.e. model input uncertainty), the computer evacuation model coupled with one arbitrary polynomial chaos expansion is used to construct the surrogate model of evacuation time, which can represent the relationship between evacuation time and input parameters. Subsequently, two-stage nested Monte Carlo simulation for the surrogate model of evacuation time can be performed, whose results can be used to determine the optimal value of the distance between exits for a certain level of probability of evacuation time. To achieve the minimum evacuation time at a certain acceptable level of probability over the design space of the distance between exits, a surrogate model-based optimization method is proposed in this paper, which can be summarized in a 4-step procedure, as shown in **Fig. 1**.

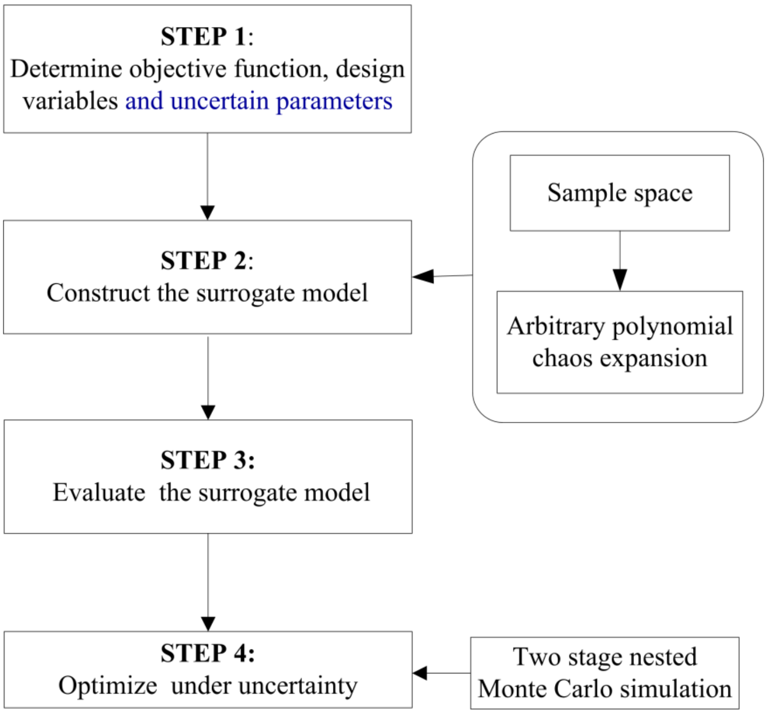

Fig. 1. Illustration of a 4-step procedure for the optimization method considering uncertainties

2.1 Objective function and design variables

Due to the randomness of fire occurrence and human behavior, in general, it is difficult to pre-determine initial values of some parameters related to occupant evacuation under fires, such as occupant density, crowd type, pre-movement time etc., which can be called as uncertain parameters. Meanwhile, geometric parameters of buildings, such as exit width, exit location, the distance between exits, etc., are generally not changed for occupant evacuation under fires. They are referred to as design variables. Evacuation time is dominated by design variables and effected by uncertain parameters. We denote evacuation time as follows.

$$T_e = f(x_{i_1}, x_{i_2}, \cdots, x_{i_{n_1}}, x_{j_1}, x_{j_2}, \cdots, x_{j_{n_2}}) \quad (1)$$

Where T_e is evacuation time; n_1 is the number of design variables associated with evacuation time; $x_{i_1}, x_{i_2}, \cdots, x_{i_{n_1}}$ are design variables, e.g. exit location, exit width, the distance between exits; n_2 is the number of uncertain parameters related to evacuation time; $x_{j_1}, x_{j_2}, \cdots, x_{j_{n_2}}$ are uncertain parameters, such as occupant density, crowd type and pre-movement times, etc.

In this paper, our purpose is to minimize evacuation time at a certain acceptable level of probability over the design space of the distance between exits. Thus, the objective function is evacuation time T_e at the level of probability p, denoted by t_e^p, and design variable is the distance between exits. The optimization problem of evacuation time under uncertainty over the design space of design variables can be formulated as follows.

Minimize: $$t_e^p = g(T_e, p) \tag{2}$$

Subject to: $x_{i_1}^{\min} \le x_{i_1} \le x_{i_1}^{\max}, x_{i_2}^{\min} \le x_{i_2} \le x_{i_2}^{\max}, \cdots, x_{i_{n_1}}^{\min} \le x_{i_{n_1}} \le x_{i_{n_1}}^{\max}$

$$x_{j_1} \sim PDF_{x_{j_1}}, x_{j_2} \sim PDF_{x_{j_2}}, \cdots, x_{j_{n_2}} \sim PDF_{x_{j_{n_2}}}$$

Where $x^{\min}$ and $x^{\max}$ are the minimum and maximum values of x; PDF_x is the probability density function of x. The cumulative distribution function of x is denoted as CDF_x. In order to find the optimal distance between exits for evacuation time under uncertainty, we assume the design variables $x_{i_1}, x_{i_2}, \cdots, x_{i_{n_1}}$ are uniformly distributed over their respective ranges. The conventional calculation procedure for the minimum t_e^p over the design spaces of variables is shown in **Fig. 2**.

According to the description in **Fig. 2**, N^2 runs are required to find the minimum t_e^p over the design spaces of variables. However, N should be large enough to achieve an accurate estimation, which leads to the number of runs N^2 tremendous. Therefore, it is computationally expensive to adopt complex computer evacuation models to find the minimum t_e^p with consideration of input parameter uncertainties. In order to save the computational cost, the next step is to adopt arbitrary polynomial chaos expansion to construct a computationally cheap surrogate model of evacuation time.

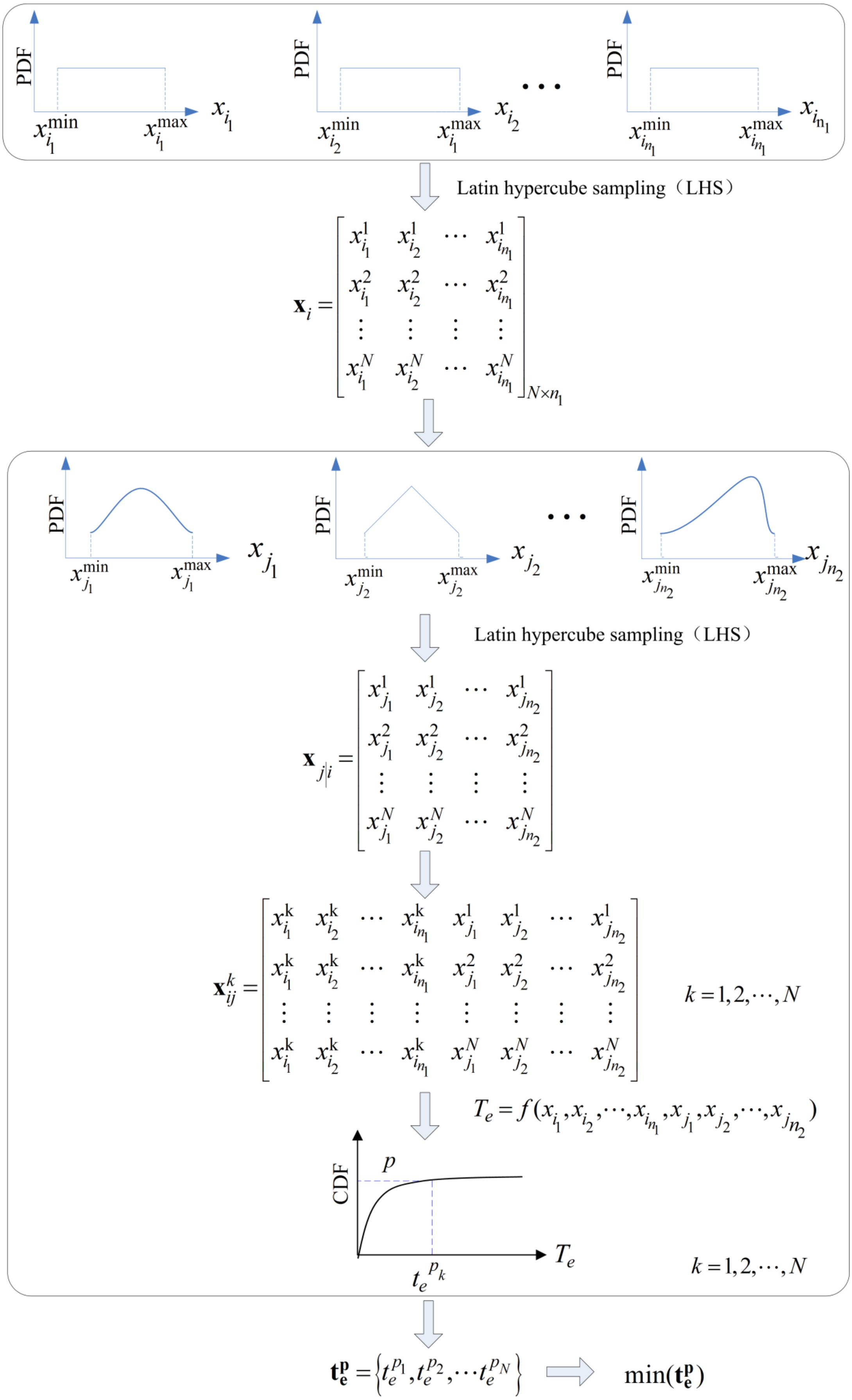

Fig. 2. Procedure for the calculation of the minimum evacuation time at a certain desired level of probability

2.2 Arbitrary polynomial chaos expansion of evacuation time

According to the work of Wiener [23], evacuation time T_e can be expressed as below.

$$T_e = \sum_{j=0}^{\infty} \alpha_j \Phi_j (x_1, x_2, \cdots, x_n) \tag{3}$$

Where α_j are arbitrary polynomial chaos expansion coefficients; Φ_j are univariate or multivariate orthogonal polynomials, which are associated with probability distributions of input parameters; n is the number of input parameters, which are divided into uncertain parameters and design variables. Uncertain parameters can be described by probability distributions, which can be determined based on buildings codes, experiment, literature, expert judgment, etc. Design variables are generally set within a certain range consistent with building codes. Moreover, these design variables are assumed to follow uniform distributions in this paper. All the input parameters of evacuation time are assumed to be independent.

To calculate evacuation time, the expansion of T_e shown in Eq. (3) need to be truncated to a certain degree d [24], which can be given as follows.

$$T_e = \sum_{j=0}^{M} \alpha_j \Phi_j (x_1, x_2, \cdots, x_n) \qquad M = \frac{(n+d)!}{n!d!}\text{-1} \tag{4}$$

Due to the independence of input parameters, the multivariable orthogonal polynomial in Φ_j is equal to the product of the corresponding one-dimensional orthogonal polynomials, which can be represented as follows.

$$\Phi_j \left(x_1, x_2, \cdots, x_n \right) = \prod_{k=1}^{n} \phi_{\beta_k^j} \left(x_k \right) \qquad j = 0,1,\cdots,M \tag{5}$$

Where β_k^j is the degree of the one-dimensional orthogonal polynomial $\phi_{\beta_k^j}\left(x_k \right)$ with the condition of $\sum_{k=1}^{n} \beta_k^j \le d$.

From the description above, one-dimensional orthogonal polynomial basis need to be determined to obtain the polynomial chaos expansion of T_e, which can be constructed by the Gram-

Schmidt orthogonalization [25]. Moreover, Oladyshkinet et al. [22] presented the relationship between the optimal one-dimensional orthogonal polynomial and statistical moments of the corresponding input parameter based on the orthogonality of polynomials, which can be given as follows.

$$\begin{bmatrix} \mu_0 & \mu_1 & \cdots & \mu_k \\ \mu_1 & \mu_2 & \cdots & \mu_{k+1} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{k-1} & \mu_{k-2} & \cdots & \mu_{2k-1} \\ 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} p_0^{(k)} \\ p_1^{(k)} \\ \vdots \\ p_{k-1}^{(k)} \\ p_k^{(k)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{6}$$

Where μ_i is the i^{th} raw moment of the input parameter; $p_j^{(k)}$ is the coefficient of the j^{th} order term for the k^{th} degree polynomial. For simplicity, in this paper the optimal one-dimensional orthogonal polynomial basis is calculated according to Eq. (6). Once the optimal one-dimensional orthogonal polynomial basis for each input parameter is determined, then orthogonal polynomials Φ_j in expansion of T_e shown in Eq. (4) can be computed according to Eq. (5).

Furthermore, unknown coefficients α_j need to be estimated for determining the polynomial chaos expansion of T_e, which can be calculated based on the non-intrusive probabilistic collocation method [26]. The optimal input sample points are the combination of the roots of the polynomial of one degree higher than that used in the polynomial chaos expansion [27]. However, for high-dimensional or high-degree situations, the number of optimal input sample points is much larger than necessary. To save the computational cost, the probabilistic collocation method [26] is used to reduce the number of input sample points and maintain the accuracy at the same time. For the probabilistic collocation method, the selection of input sample points is based on probability distributions of input parameters, which means input sample points in high-probability regions are given high priorities. The probabilistic collocation method is given as follows: Firstly, optimal sample points of input parameters are ranked according to their probability distributions. The iterative computation for the information matrix related to input samples is performed until the rank of the information matrix is equal to the number of input samples [24], which indicates that the linear equation of unknown polynomial chaos expansion coefficients α_j is solvable. Subsequently, the required input sample points with the minimum number are available. Once the necessary input sample points are determined, the corresponding output sample points can be generated by running computer evacuation models, such as buildingEXODUS [9], Simulex [10,11], FDS+Evac [1] and Pathfinder [12]. While the input and output samples are obtained, unknown coefficients α_j can be computed through the Singular Value Decomposition (SVD) algorithm.

Thus, the polynomial chaos expansion of evacuation time with any degree can be constructed according to the above description.

2.3 Evaluation of the Surrogate Model

Based on the arbitrary polynomial chaos expansion, the surrogate model of evacuation time can be constructed. However, the accuracy of the surrogate model need to be evaluated. There are various metric methods used to evaluate the accuracy of the model, such as residual error, mean square error (MSE), root mean square error (RMSE), coefficient of determination (R^2), predicted R^2, maximum absolute error (MAE), relative maximum absolute error (RMAE), relative average absolute error (RAAE), cross-validation and so on [28]. In order to eliminate the effect of the unit, the average relative error $\overline{e}$ is used as a metric of the accuracy for the surrogate model of evacuation time, which is described as follows.

$$\overline{e} = \frac{1}{N}\sum_{i=1}^{N} e^{(i)} \qquad e^{(i)} = \left| \frac{\hat{t}_e^{(i)} - t_e^{(i)}}{t_e^{(i)}} \right| \tag{7}$$

The lower the average relative error $\overline{e}$ is, the greater the accuracy demanded for the surrogate model of evacuation time is, which lead to need to run a larger number of evacuation simulations. In general engineering applications, the acceptable relative error is commonly assumed to be 0.05, 0.1, or 0.15 [29]. However, the calculation of evacuation time is of great concern in the fire safety assessment and design of buildings. Thus, to obtain a more accurate surrogate model of evacuation time, the acceptable average relative error is selected to be 0.5% in this paper, which is less than 0.05.

2.4 Two-stage nested Monte Carlo simulation

Once the accuracy of the surrogate of evacuation time is acceptable, the next step is to determine the optimal distance between exits for a certain desired level of probability of evacuation time. In order to determine the optimal distance under uncertainties, the most straightforward method is two-stage nested Monte Carlo simulation of the surrogate model of evacuation time. The surrogate model of evacuation time constructed based on the arbitrary polynomial chaos expansion is a combination of orthogonal polynomials. Thus, it is computationally affordable to perform a two-stage nested Monte Carlo simulation of the surrogate model of evacuation time, and then the optimal distance between exits under certainties can be easily found from the simulation results. To balance the resulted accuracy and the computational cost, the two stage nested Monte Carlo simulation of surrogate model of evacuation time is a suitable solution for determining the optimal distance between exits under uncertainties in this paper.

There are two loops in a two-stage nested Monte Carlo simulation, i.e. the outer loop and inner loop. Design variables are sampled in the outer loop and uncertain parameters are sampled in the inner loop. Firstly, the sample space of the distance between exits is generated by the Latin hypercube sampling (LHS) [30] in the outer loop. And then, the LHS method is used to obtain the sample space of uncertain parameters given the 1st sample of the distance between exits. And then run the surrogate model of evacuation time from the 1st sample to the last sample of uncertain parameters given the 1st sample of the distance between exits. Afterwards, the conditional probability distribution of evacuation time is calculated given the 1st sample of the distance between exits. Then the 2nd sample of the distance between exits is taken from its sample space, and the Monte Carlo simulation of the surrogate model of evacuation time is performed for the LHS sample space of uncertain parameters given the 2nd sample point of the distance between exits. And based on the results of statistical analysis, the conditional probability distribution of evacuation time given the 2nd sample of the distance between exits can be obtained. Repeat the above steps until every sample of the distance between exits is taken, and the conditional probability distribution of evacuation time given the distance between exits can be generated. Finally, based on the results from the two-stage nested Monte Carlo simulation of the surrogate model of evacuation time, the optimal distance between exits for a certain desired probability level of evacuation time can be easily determined. The procedure for a two-stage nested Monte Carlo simulation of the surrogate model of evacuation time is shown in **Fig. 3**.

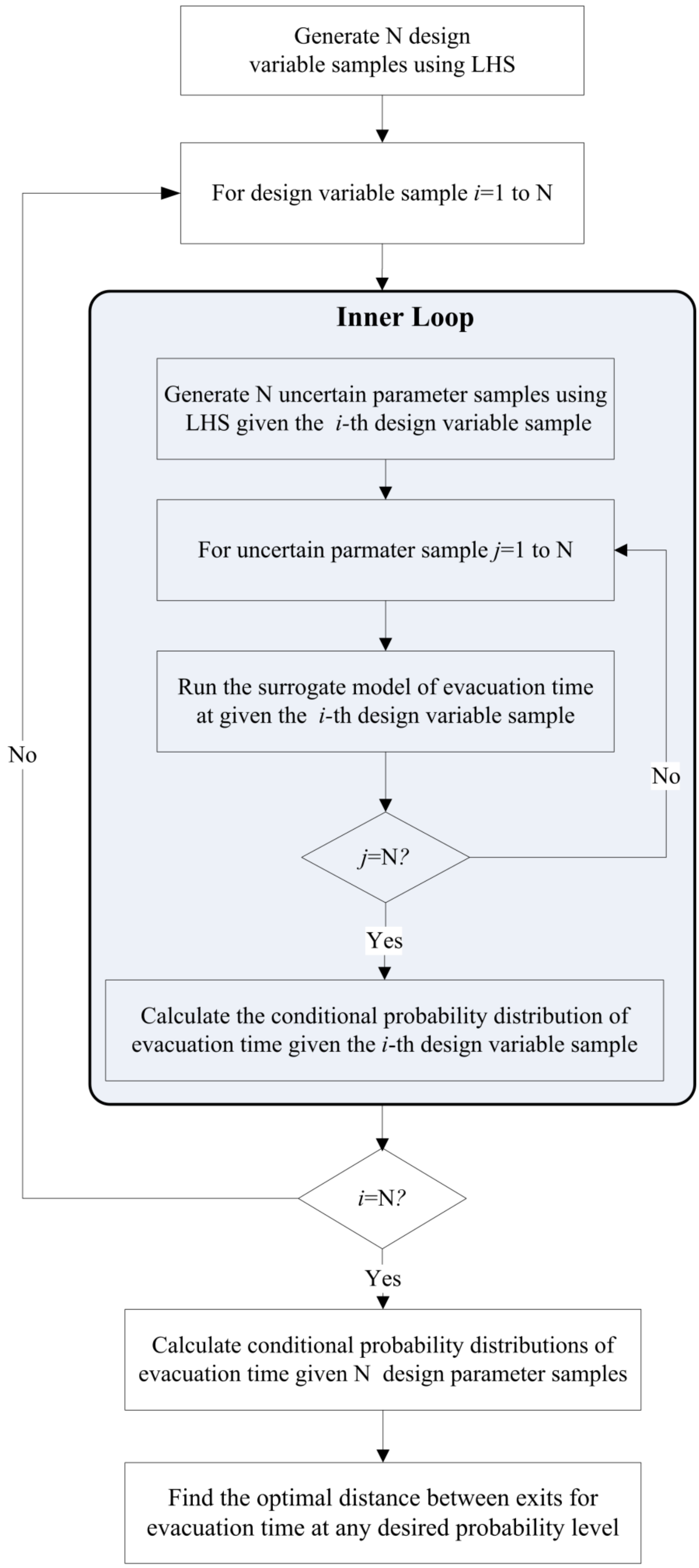

Fig. 3. Two-stage nested Monte Carlo simulation for the surrogate model of evacuation time

3. Case study and analysis

3.1 Case description

In order to demonstrate the proposed optimization method, one single-room fire compartment of public buildings is presented in this paper. According to the rule with regards to the fire compartment in the Chinese code GB 50016-2014 [2], the maximum floor area is 2500 m^2 and the minimum number of exits/doors is 2 for a single room of public buildings. Considering a worse situation, the floor area of this fire compartment is set to be 2500 m^2 with 2 exits denoted by Exit A and Exit B. Besides, obstacles are not presented in this fire compartment, which eliminates the effect of the room layout. Moreover, for each evacuation scenario, occupants are assumed to be randomly distributed in the whole building space, which removes the influence of uneven spatial distributions of occupants. And the floor plan of this fire compartment is shown in **Fig. 4**.

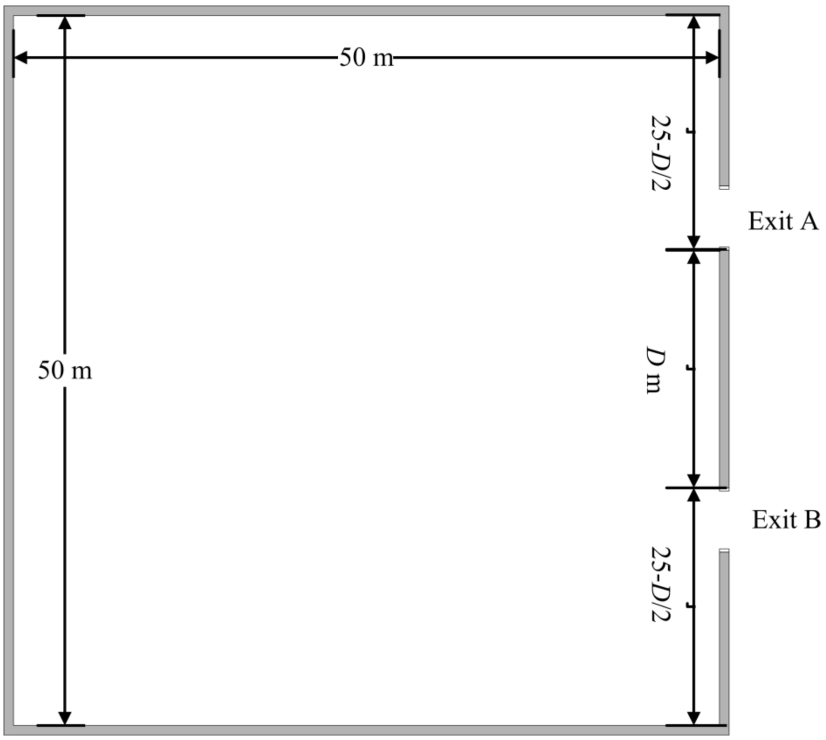

Fig. 4. Floor plan of one single-room fire compartment used in this case

Due to the complexity and randomness of fire and occupant characteristics, there are various parameters related to crowd evacuation simulation, which can be classified into fire characteristic parameters, building characteristic parameters and human characteristic parameters [31]. Fire characteristic parameters include the location of fire source, fire growth rate, smoke yield, toxicity, heat, temperature, etc., which may have significant influences on occupant behaviors and other enclosures (e.g. availability of the door/exit itself). Building characteristic parameters are associated with building geometry, building layout, exit width, exit location, the distance between exits, etc. Human characteristic parameters contain occupant density, crowd type, occupant walking velocity, occupant body size, pre-movement time, knowledge and experience, social features, familiarity with layout, etc. In this paper, we assume that occupants evacuate without the effects of fires before the occurrence of untenable conditions. Thus, fire characteristic parameters are not considered in this case. Albrecht et al. [32] suggested that pre-movement time and occupant density are the most influential parameters for the egress simulation. Due to the randomness of human behavior and decision, pre-movement time of each individual may be different, which may alleviate the congestion around the exit. And with consideration of ideal conditions of emergency evacuation, pre-movement times of occupants are set to 0 s in this case, i.e., occupants evacuate immediately once the alarm initiates a fire signal. From the point of congestion, the assumption that evacuees with 0 s pre-movement times may be more conservative. However, due to the complexity and randomness of evacuation scenarios (e.g. initial location of individual occupant, walking speed, occupant load, etc.), the assumption of 0 s pre-movement time as a conservative choice is not true in general. Besides, Children are particularly vulnerable to fires, which should be focused on in fire protection design of buildings. Based on the above analysis, uncertainties in occupant density and the child-occupant load ratio are considered in this case study. Actually, the impacts of different variables are scenario-dependent. For example, due to mobility impairments, people with disabilities are crucial to evacuation in hospitals. For complex hotels, occupants may not be familiar with the environment [33], which will result in the familiarity with the exits having a significant effect on evacuation. However, for apartments, occupants are familiar with the environment. Therefore, the familiarity with the exit may have little effect on evacuation in apartments.

The child-occupant load ratio is 0.7 for schools [11], which is obviously a worse circumstance for public buildings. In this case, the mean of the child-occupant load ratio is assumed to be 0.7. For occupant density, the Chinese code GB50016-2014 [2] assumes the value of 1.0 persons/m^2 for exhibition halls, which is typical of public buildings. Thus, the mean of occupant density is taken as 1.0 persons/m^2 in this case. Due to the scarce available data on stochastic variables of occupant evacuation, the standard deviations of uncertain parameters are usually assumed to be 10-20% of their mean values [34]. Herein, the standard deviations of the child-occupant load ratio and occupant density are selected to be the 20% and 10% of their mean values. However, there is a lack of evacuation data and the normal distribution is the most common type of distributions. Therefore, occupant density and the child-occupant load ratio are assumed to be truncated normal distributions in this case study, which are located in the range of 1 and 2 standard deviation away from their mean values, as shown in **Table 1**.

Table 1 Distributions of uncertain parameters for the fire compartment described above

Uncertain parameters	Distribution type	Mean value	Standard deviation	Range
Occupant density/ (persons/m$^{2)}$	Normal	1.0	0.2	0.8-1.2
Child-occupant load ratio	Normal	0.7	0.07	0.56-0.84

Exit width is generally determined based on the number of evacuees according to building codes. For example, the minimum exit width per 100 persons is set to be 0.65 m for most public buildings according to the Chinese code GB50016-2014 [2]. Here, considering a worse situation, exit width per 100 persons is taken as 0.1 m in this case. Thus, the total exit width in this case is taken as 2.5 m, which is the product of the floor area 2500 m^2, the mean occupant density 1.0 persons/m^2 (see **Table 1**) and exit width per person 0.001 m/person. Meanwhile, Exit A and Exit B are assumed to be the same size 1.25 m, which are symmetrically arranged along the center of the wall derived from the work of Zhao et al.[15]. Through the above discussion, the distance between exits (D) is set in the range of 0-47.5 m (see **Table 2**) in this case.

Table 2 Design parameters in this case

Design parameters	Range
Exit separation distance, D(m)	0-47.5

According to the proposed optimization method under uncertainties, the arbitrary polynomial chaos expansion of T_e should be repeatedly constructed until the surrogate model of evacuation time is accurate enough. And then a two-stage nested Monte Carlo simulation is applied to the surrogate model of evacuation time. Based on the results from the two-stage nested Monte Carlo simulation for the surrogate model of evacuation time, the optimal distance between exits for a certain desired level of probability of evacuation time can be easily found.

The output samples (i.e. evacuation time samples) for constructing the arbitrary polynomial chaos expansion of evacuation time can be generated by running computer evacuation models. There are various computer evacuation models, such as buildingEXDOUS, Simulex, Pathfinder, etc. For the FDS+Evac computer model, a great number of input files can be automatically generated by adopting the MATLAB codes to modify the base input file. Besides, evacuation time data can also be automatically extracted from the output files of the FDS+Evac model using the MATLAB codes. Thus, the FDS+Evac model is used here to predict evacuation time samples. Furthermore, attributes of each occupant are randomly generated from the predefined database, and in the given building space in the FDS+Evac model. Thus, evacuation time is different for repeated simulations of the same evacuation scenario using the FDS+Evac model. In order to obtain a representative evacuation time, the appropriate number of repeated simulations should be determined for the same evacuation scenario. Korhonen et al. [1] adopted 5 simulations with the FDS+Evac model for the same sports hall evacuation scenario. To be prudent, in this paper each evacuation scenario repeatedly runs 10 times with the FDS+Evac model and the average of 10 evacuation times is used as T_e for this evacuation scenario. However, to achieve a more accurate estimation, functional analysis and central limit theorem should be used to determine the appropriate number of repeated simulations for each evacuation scenario [13], which will be adopted in our future work.

3.2 Results and discussion

In order to obtain accurate enough surrogate model of evacuation time, the 2nd and 3rd degree arbitrary polynomial chaos expansions of evacuation time are constructed with 16 and 41 sample points, whose input parameters are occupant density, the child-occupant load ratio and the distance between exits (D) (see **Tables 1** and **2)**. Then, the Monte Carlo simulation technique is applied to the 2nd and 3rd degree polynomials to obtain the corresponding cumulative distribution functions (CDFs) of evacuation time T_e, as shown in **Fig.5**.

Afterwards, the average relative error $\overline{e}$ is used to measure the difference between two CDFs of evacuation time T_e. The $\overline{e}$ between two CDFs from the 2nd and 3rd degree polynomial chaos expansions is 0.76%, which is larger than 0.5%. Thus, the 4th degree polynomial chaos expansion of T_e is constructed in this case. The $\bar{e}$ between two CDFs from the 3rd and 4th degree polynomial chaos expansions is 0.33%, smaller than 0.5%. Thus, the 4th degree polynomial chaos expansion of T_e can be used as the surrogate model of evacuation time in this case, which requires 86 sample points.

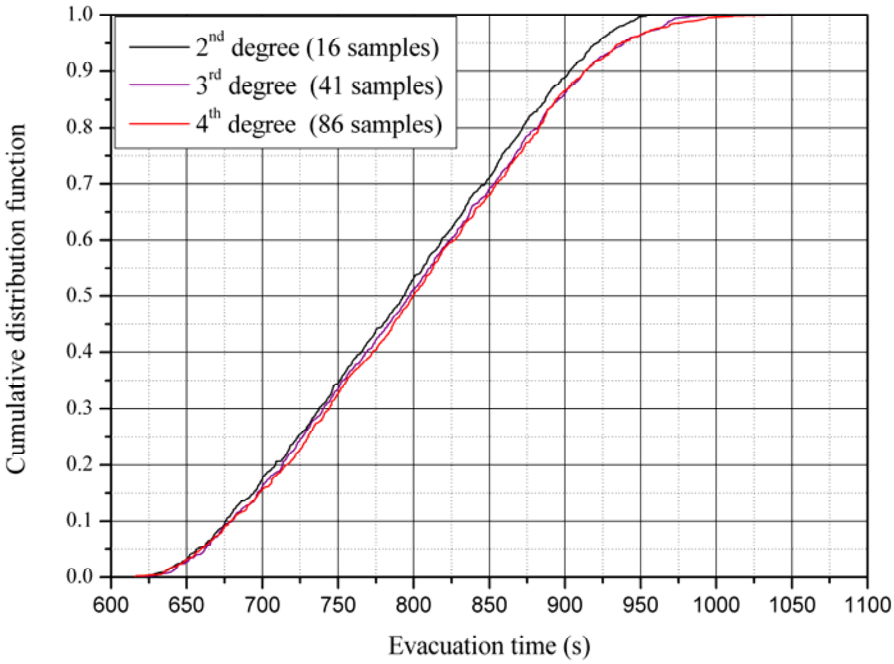

Fig. 5. Cumulative distribution functions (CDFs) of evacuation time from the Monte Carlo simulations of the 2nd, 3rd and 4thdegree polynomial chaos expansions

Additionally, in order to find the optimal distance between exits for a certain desired level of probability of evacuation time, a two-stage nested Monte Carlo simulation is applied to the surrogate model of evacuation time in this case. Based on the two-stage nested Monte Carlo simulation for the 4th polynomial chaos expansion of evacuation time, **Fig.6** gives the minimum evacuation time and the optimal distance between exits for different levels of probability.

Fig.6 shows that the minimum evacuation time increases significantly along with the level of probability. For example, while the levels of probability are 80% and 95%, the minimum evacuation times are 863 s and 910 s. Thus, the acceptable level of probability is crucial for the selection of evacuation times, which should be prudently determined in fire risk assessment and performance-based fire protection design of buildings.

Moreover, according to the rule of minimizing the maximum travel distance, the optimal distance between exits (D) is 23.8 m. However, the optimal distance between exits varies with the level of probability of evacuation time, which is away from 23.8 m in this case, as shown in **Fig. 6**. While occupant density and the child-occupant load ratio are in the ranges of 0.8-1.2 persons/m^2 and 0.56-0.84 with the narrow exit width (2.5m), there may be significant interactions between occupants and occupants as well as occupants and buildings during the evacuation process, especially around the exits. Thus, evacuation time of each occupant cannot be simply calculated by the travel distance divided by the unimpeded walking speed. A high level of probability of evacuation time indicates a high occupant density and/or a large child-occupant load ratio in this case. Due to the interaction forces between occupants and occupants as well as occupants and buildings varying with occupant density and/or the child-occupant load ratio, the optimal distance between exits will be different for different levels of probability, as shown in **Fig. 6**. For complex buildings, it can be speculated that the difference in the optimal distance between exits at different levels of probability will be much more significant. Besides, from **Fig. 6** it can also be seen that while the level of probability of evacuation time is lower than 32%, the optimal distance between exits is nearly unchanged around 14 m. However, while it is larger than 32%, the optimal distance between exits sharply increases to around 30 m, which suggests that the level of probability 32% is a critical point due to the transition from sparse crowd to dense crowd. However, for different distributions, the outcome may be significantly different and the critical point may be located in a very different place. Thus, the distributions of uncertain parameters should be carefully determined.

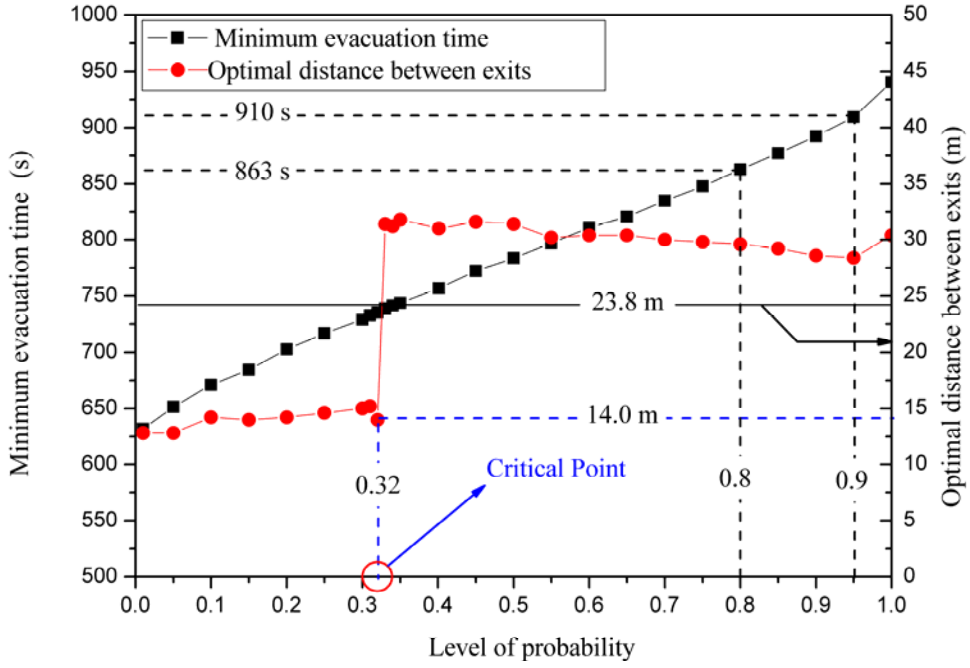

Fig. 6. Minimum evacuation time and optimal distance between exits versus the level of probability

Furthermore, **Fig. 7** gives evacuation time at different levels of probability along with different distances between exits based on the results from the two-stage nested Monte Carlo simulation for the surrogate model of evacuation time. For a certain value of *ASET*, the acceptable distance between exits can be determined easily. For example, while *ASET* is 950 s and the acceptable level of probability is 99.9%, the distance between exits (*D*) can be selected in a wide range of 12.5-37.5m, as shown in **Fig.7**.

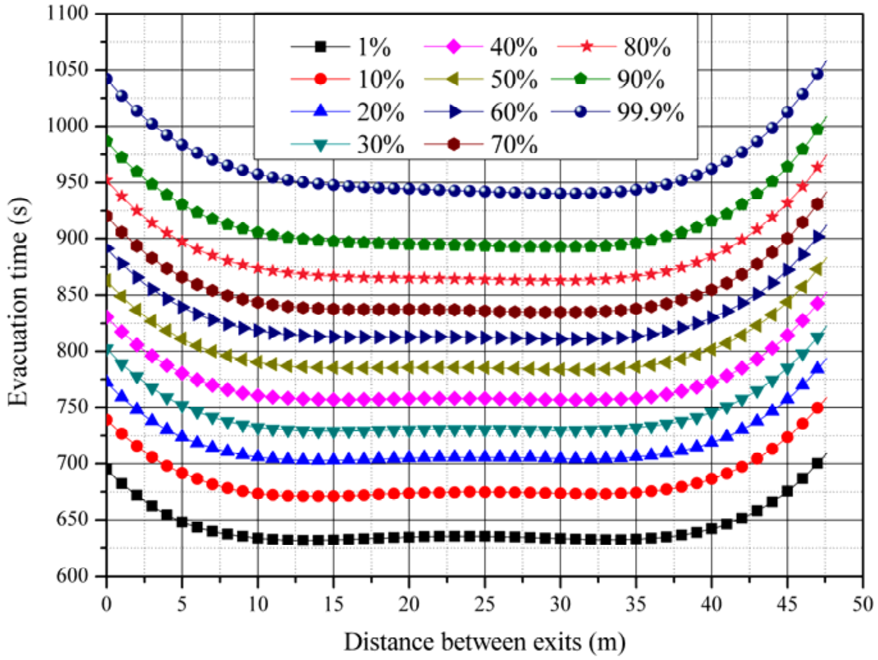

Fig. 7. Evacuation time at different levels of probability versus the distance between exits

Finally, in order to verify the accuracy of the surrogate model of evacuation time T_e, the CDF of evacuation time from the Monte Carlo simulation of the FDS+Evac model is also provided (see **Fig. 8**). The value of $\bar{e}$ between the CDFs of evacuation time from the 4th degree polynomial chaos expansion and the Monte Carlo simulation of the FDS+Evac model is 0.37%, which indicates that the precision of the surrogate model of evacuation time is similar to the FDS+Evac model. Therefore, the 4th degree polynomial chaos expansion of T_e can be used as the surrogate model of evacuation time in this case, whose computational cost (86 samples) is reduced significantly compared with the Monte Carlo simulation of the FDS+Evac model (1000 samples), as shown in **Fig. 8**.

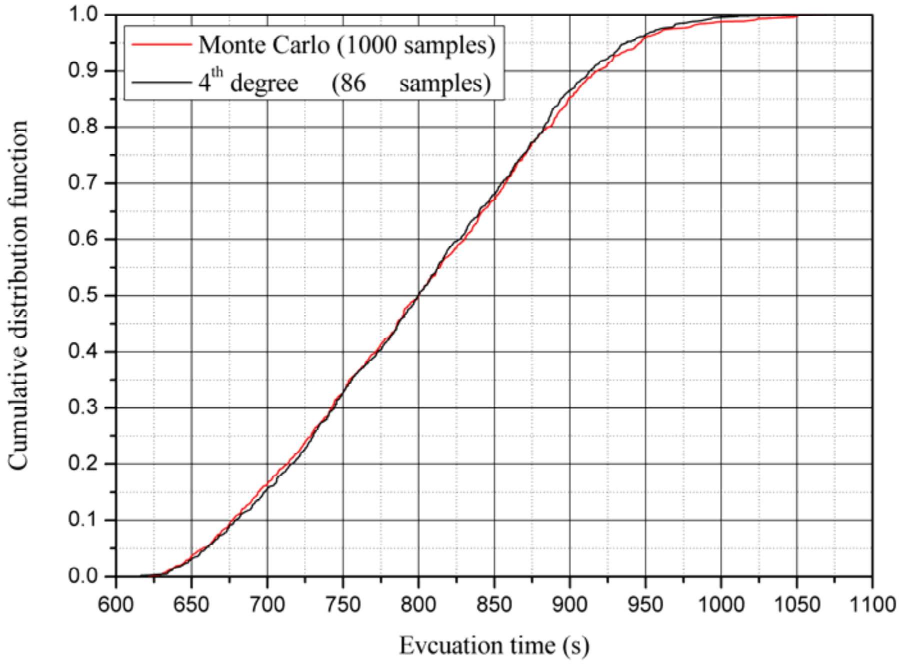

Fig. 8. Comparison of the accuracy of the surrogate model of evacuation time and the FDS+Evac model for uncertainty quantification of evacuation time

4. Conclusions and future work

In this paper, a surrogate-based optimization method, which couples the arbitrary polynomial chaos expansion of evacuation time with a two-stage nested Monte Carlo simulation, is proposed to find the optimal distance between exits for evacuation time under uncertainties with a significantly reduced computational cost. In order to illustrate the proposed method, one single-room fire compartment with 2 exits is presented with consideration of uncertainties in occupant density and the child-occupant load ratio. Based on this case study, some conclusions can be drawn as follows.

Compared with the conventional optimization method under uncertainties, i.e. the two-stage nested Monte Carlo simulation or the coupling of numerical optimization techniques to the Monte

Carlo simulation of complex computer evacuation models, the computational cost of the proposed method is significantly reduced.

Moreover, the optimal distance between exits changes with the level of probability of evacuation time, which is also away from that determined by minimizing the maximum travel distance. Besides, for different levels of probability of evacuation time, the difference in the optimal distance between exits is probably more significant in complex buildings. Meanwhile, there is a critical level of probability for the transition of the optimal distance between exits.

Furthermore, the minimum evacuation time over the distance between exits sharply increases with the level of probability. Thus, it is necessary to establish a criterion for the acceptable level of probability in fire protection design of buildings. Meanwhile, for a certain *ASET*, it is easy to give a suitable range of the distance between exits based on the results from the two-stage nested Monte Carlo simulation for the surrogate model of evacuation time.

However, there are still some limitations in this paper. Firstly, the case studied is a simple single room with consideration of occupant density and the child-occupant load ratio having truncated normal distributions. For different distributions of uncertain parameters, the outcome may be significantly different and the critical point may be located in a very different place.

Secondly, we don't have rigorous analysis on the number of repeated simulations for each evacuation scenario. Based on functional analysis, Ronchi et al. [13] propose a quantitative method to determine the appropriate number of repeated simulations for every evacuation scenario, which will be adopted in our future work.

Thirdly, the proposed method is only feasible to handle independent input parameters, which cannot be used to deal with correlated parameters. For correlated parameters, the independent orthogonal polynomial basis need to be constructed, and detailed description is presented in the work of Navarro et al. [35].

Besides, the familiarity of occupants with exits and the impacts of fire on occupant evacuation are not considered in this paper, which will be focused on in our future work.

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