An Arbitrary Polynomial Chaos-Based Approach to Analyzing the Impacts of Design Parameters on Evacuation Time under Uncertainty

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ABSTRACT

In performance-based fire protection design of buildings, much attention is paid to design parameters by fire engineers or experts. However, due to the time-consuming evacuation models, it is computationally prohibitive to adopt the conventional Monte Carlo simulation (MCS) to examine the effect of design parameters on evacuation time under uncertainty. To determine the suitable values of design parameters under uncertainty with the reduced significantly computational cost, an arbitrary polynomial chaos-based method is presented in this paper. Arbitrary polynomial chaos expansion is used to construct surrogate models of complex evacuation models. Afterwards, simple analytical method can be adopted to calculate the mean and standard deviation of evacuation time as well as Sobol sensitivity indices based on the arbitrary polynomial chaos coefficients. Meanwhile, the distribution of evacuation time can be generated by coupling Latin hypercube sampling (LHS) to the surrogate model. To demonstrate the proposed method, a case in accordance with the Chinese code GB50016-2012 is presented, evaluating the impact of exit width on evacuation time under arbitrary uncertainty caused by occupant density and child-occupant load ratio in a single-storey fire compartment with two exits. And the results of this case show that the proposed method can achieve the distribution of evacuation time close to those from MCS while dramatically reducing the number of evacuation simulations. When exit width per 100 persons is designed in the range of 0.1m and 0.5m, evacuation time uncertainty is severely affected by exit width and is more significant in exit with smaller width. However, exit width has little effect on Sobol sensitivity indices, the reliability level of a certain safety factor, and safety factor at a certain reliability level.

KEYWORDS: performance-based fire protection design, egress, evacuation time, uncertainty analysis, risk assessment, statistics

INTRODUCTION

Occupant evacuation is extremely complex and is influenced by the physiological, psychological, and sociological aspects of evacuees and external environment [1]. Due to the randomness of fire occurrence and the variability of occupants' characteristics, there exist many uncertain factors, such as pre-movement time, occupant density, occupant type, familiarity with exits, and occupant mobility etc. Thus, evacuation time is highly uncertain for a specific building. Traditionally, prescriptive fire protection design codes adopt safety factors to deal with uncertain factors related to crowd evacuation [2]. However, with the help of prescriptive building fire protection codes, there is still a lack of specific guidance about the selection of safety factors. Meanwhile, it is also difficult to determine the safety performance of buildings and optimal design parameters using prescriptive building codes. Thus, in order to address the problems above, the arbitrary polynomial chaos- based method is proposed in this paper.

During the last few decades, research has been widely conducted on the effect of design parameters on evacuation time. Based on the results of Monte Carlo simulation (MCS), Meacham et al. [3] suggested that significant uncertain parameters for evacuation time may be associated with the geometry of buildings. Fang et al. [4] employed a multi-grid evacuation model to investigate the effect of door width on evacuation time and assumed that door width has a more significant influence on evacuation time in fire situations than normal conditions. Zhao et al. [5] adopted a cellular automata evacuation model to conduct the performance-based design of building exits in deterministic scenarios and suggested that the layout of

exits should be symmetrical and the distance between exits should be designed using optimization techniques. In order to improve building designs for a certain deterministic scenario with the reduced computational cost, Tavares et al. [6] combined the Building EXODUS evacuation model and design of experiments to obtain the deterministic response surface of evacuation time. Afterwards, numerical optimization methods are applied to determine the optimal design parameter. In current building regulations, exit locations are usually determined by the maximum travel distance. In order to find an alternative method to determine exit locations in a certain specified scenario, Tavares et al. [7] compared evacuation times from the exit location determined by the relative distance between exits and the maximum travel distance through a case study, which suggested that evacuation time from the exit location defined by the relative exit width is smaller and the exit location can be determined by the relative distance between exits. It can be seen that Ref. [6] and [7] focus on the optimization of design parameters in deterministic values of uncertain parameter using the deterministic response surface method and the comparison of deterministic evacuation simulations, and thus the optimization of the design parameters under uncertainty using the stochastic response surface method are paid attention in this paper. Furthermore, in order to compute the reliability of evacuating successfully with the low computational cost, Cornelius et al. [8] adopted the adaptive deterministic response surface method to build the surrogate models of fire and evacuation simulations.

As the discussion above, the impacts of design parameters on evacuation time are mainly studied in the case of assigning deterministic values to uncertain parameters such as occupant density. However, evacuation time is highly uncertain due to the randomness of fire occurrence and human characteristics. Meanwhile, due to the complexity of evacuation models, it is computationally expensive to adopt the conventional MCS to investigate the effect of design parameters on evacuation time under uncertainty. Stochastic response surface methods can be used to deal with the output uncertainty caused by the uncertainty in input parameters with the low computational cost [9]. And the arbitrary polynomial chaos proposed by Witteveen et al. [10] is one of the stochastic response surface methods, which can handle arbitrary distributions of input parameters only with limited statistical moments. Thus, in order to take the reduced significantly computational cost to analyze the effect of design parameters on evacuation time under arbitrary uncertainty, the arbitrary polynomial chaos-based method, is proposed here. Based on results of computationally expensive evacuation models, arbitrary polynomial chaos expansion can be used to construct surrogate models of evacuation time with uncertain input parameters. Then, the mean value and standard deviation of evacuation time as well as Sobol sensitivity indices can be obtained analytically based on the arbitrary polynomial chaos coefficients. Furthermore, Latin hypercube sampling (LHS) can be applied to uncertain parameters to generate input samples, and then run the surrogate model of evacuation time to generate the distribution of evacuation time.

ARBITRARY POLYNOMIAL CHAOS -BASED APPROACH

The arbitrary polynomial chaos-based approach, which is used to investigate the effect of design parameters on evacuation time under uncertainty with reduced significantly computational cost, whose flowchart can be described in Fig.1.

Evacuation Models and Input Parameters

Evacuation time can be calculated by simple calculations or complex computer evacuation models [11]. Simple calculations assume a number of simplifications in the evacuation process, which are based on observations and evacuation experiments. In order to obtain more accurate results, complex computer evacuation models were developed. According to different classification methods, computer evacuation models can fall into microscopic and macroscopic models; discrete and continuous models; deterministic and stochastic models; rule-based and force-based models; high and low fidelity models [12]. Zheng et al. [13] reviewed seven methodological approaches for crowd evacuation and suggested that different modeling approaches should be combined together to simulate occupant evacuation. Ronchi et al. [14] analyzed the advantages and limitations of computer evacuation models and indicated that computer evacuation models should be based on reasonable modeling assumptions, embedded sub-algorithms and treatment of model uncertainty. FDS+Evac computer model is an agent-based social force model, which is also a stochastic evacuation model. Moreover, FDS+Evac results have been compared with some other evacuation models and the simulation results of evacuation time are quite similar [15]. Thus, evacuation

time samples that are necessary for constructing the surrogate model of evacuation time can be obtained from simulation results of the FDS+Evac model in this paper.

Input parameters of evacuation models can be classified into two types: design parameters and uncertain
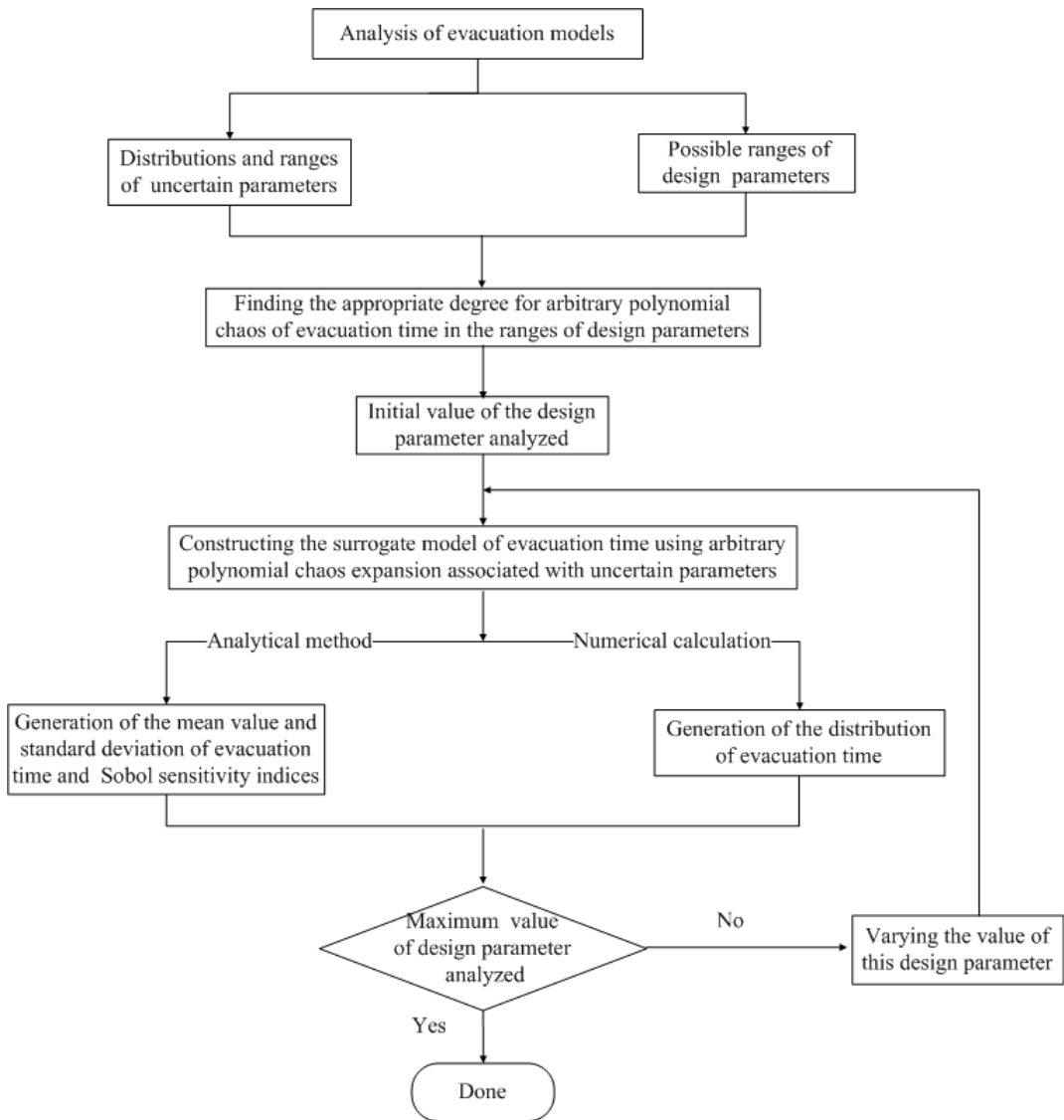

Fig. 1. Procedure for effects of design parameters on evacuation time under arbitrary uncertainty

parameters. For the performance-based fire protection design of buildings, design parameters are generally related to the building geometry, which contains exit width, exit location and the number of exits. It is expensive to change these design parameters once buildings are built. Thus, design parameters should be chosen prudently by fire protection engineers or experts. Generally the ranges of design parameters can be assumed according to the building codes, building geometry or occupancy type. Since there is a high degree of uncertainty in fire occurrence, it is difficult to determine the initial values of some parameters associated with crowd evacuation such as occupant density, occupant type and so on, which are considered as uncertain parameters due to the incomplete knowledge. Generally, the distributions of uncertain parameters can be assumed based on the literature, observations, codes and recommendations by experts [16]. Through the analysis above, it can be seen that evacuation time is determined by design parameters and influenced by uncertain parameters, whose uncertainty should be quantified.

Arbitrary Polynomial Chaos Expansion

Arbitrary polynomial chaos is the combination of polynomial chaos expansion and Gram-Schmidt orthogonalization [10], which can be used to address the output uncertainty affected by uncertain parameters with arbitrary distributions, such as truncated standard distributions and histogram distributions etc. The predictive model of evacuation time can be expressed as follows.

$$T_e = F(\mathbf{x}) \qquad\qquad \mathbf{x} = \{x_1, x_2, \cdots, x_n\} \tag{1}$$

Where T_e is evacuation time; n is the number of uncertain parameters; $\mathbf{x} = \{x_1, x_2, ..., x_n\}$ is the vector set of independent uncertain parameters of evacuation models.

According to Wiener [17], the output T_e can be expressed in mathematical series with regard to independent input variables $x_1, x_2, \cdots, x_n$, as shown in Eq. 2.

$$
\begin{aligned}
T_e(x_1, x_2, ..., x_n) = &\, \alpha_0 + \sum_{i1=1}^{n} \alpha_{i1}\psi^1(x_{i1}) + \sum_{i1=1}^{n} \alpha_{i1,i1}\psi^2(x_{i1}) + \sum_{1 \le i1 < i2 \le n} \alpha_{i1,i2}\psi^1(x_{i1})\psi^1(x_{i2}) + (\sum_{i1=1}^{n} \alpha_{i1,i1,i1}\psi^3(x_{i1}) \\
&+ \sum_{1 \le i1 < i2 \le n} \alpha_{i1,i1,i2}\psi^2(x_{i1})\psi^1(x_{i2}) + \sum_{1 \le i1 < i2 \le n} \alpha_{i1,i2,i2}\psi^1(x_{i1})\psi^2(x_{i2}) + \sum_{1 \le i1 < i2 < i3 \le n} \alpha_{i1,i2,i3}\psi^1(x_{i1})\psi^1(x_{i2})\psi^1(x_{i3})) + ...
\end{aligned}
\tag{2}
$$

Where the coefficients α are unknown, which are used to quantify the contribution of individual uncertain parameters or the interactions between/among uncertain parameters to the uncertainty of T_e; ψ is the optimal orthogonal basis for individual input parameters, whose superscript indicates the degree.

For computational purposes, the Eq. 2 should be truncated with a certain degree d [18], as shown in Eq. 3.

$$
\begin{aligned}
T_e(x_1, x_2, ..., x_n) \approx &\, \alpha_0 + \sum_{i1=1}^{n} \alpha_{i1}\psi^1(x_{i1}) + \sum_{i1=1}^{n} \alpha_{i1,i1}\psi^2(x_{i1}) + \sum_{1 \le i1 < i2 \le n} \alpha_{i1,i2}\psi^1(x_{i1})\psi^1(x_{i2}) + \cdots + \\
&(\sum_{i1=1}^{n} \alpha_{i1,i1,\cdots,i1}\psi^d(x_{i1}) + \sum_{1 \le i1 < i2 \le n} \alpha_{i1,\cdots,i1,i2}\psi^{d-1}(x_{i1})\psi^1(x_{i2}) + \cdots)
\end{aligned}
\tag{3}
$$

Where the number of unknown coefficients α can be determined by n and d, which is equal to $\dfrac{(n+d)!}{n!d!}$.

The optimal orthogonal basis ψ can be determined according to the Gram-Schmidt orthogonalization [10] Based on Gram-Schmidt orthogonalization, Oladyshkin [19] gave the relationship between optimal orthogonal basis and statistical moments of uncertain input parameters, whose distributions can be arbitrary with limited statistical moments, and the main steps to obtain the optimal orthogonalization basis are summarized as follows:

Firstly, for ease of analytical development, all uncertain parameters x_i are transformed linearly to x_i' with 0 mean value and unit variance as follows.

$$x_i' = \frac{x_i - E(x_i)}{\sqrt{\operatorname{var}(x_i)}} \qquad\qquad i = 1, 2, \cdots, n \tag{4}$$

Where $E(x_i)$ and $\operatorname{var}(x_i)$ $\operatorname{var}(x_i)$ are the mean and variance of x_i.

Secondly, the optimal orthogonal basis $\psi(x_i')$ can be generated by solving the Eq. (14) in Ref. [19], which involves the statistical moments of x_i';

Finally, for more useful properties, the obtained the j-th degree optimal orthogonal basis $\psi^j(x_i')$ can be normalized as bellows.

$$\phi^j(x_i') = \frac{\psi^j(x_i')}{\int \psi^j(x_i')^2 \, p(x_i') dx'} \qquad\qquad i = 1, 2, \cdots, n; \; j = 1, 2, \cdots, d \qquad\qquad (5)$$

Where $\phi^j(x_i')$ is the j-th degree normalized orthogonal basis for x_i'; $p(x_i')$ is the probability density function for x_i', which is identical with $p(x_i)$.

Determination of Sample Points

Unknown coefficients α can be solved using the regression methods [18], which is associated with sample points of the input and output. For a certain uncertain input parameter, the optimal sample points are the roots of the orthogonal polynomial of one degree higher than that used in the expansion [20]. For the d-th degree expansion with n uncertain input parameters, the number of the optimal input sample points is $(d+1)^n$, which may be much larger than necessary. In order to avoid the unnecessary simulations and assure the accuracy, Isukapalli et al. [9] proposed two methods to select necessary input samples from the set of optimal input samples, i.e. selecting randomly and selecting based on probability distributions. For the former, the input sample size is recommended to be twice as many as the number of unknown coefficients α. For the latter, the optimal input samples in high probability regions are given high priorities. In this paper, the latter is adopted to determine the necessary input sample points. Afterwards, the FDS+Evac model is performed on the necessary input sample points to generate the corresponding T_e sample points. Once the input and output sample points are obtained, unknown coefficients α can be acquired by solving Eq. (3) using the singular value decomposition algorithm.

Determination of the Appropriate Degree for the Surrogate Model of Evacuation Time

When the optimal normalized orthogonal basis ϕ and unknown coefficients α are obtained, the d-th degree polynomial representation of T_e can be determined. However, it is necessary to verify the degree for T_e with the purpose of assuring the quality of the results.

In the range of a certain design parameter, the determination of the suitable degree for T_e can be described as follows: firstly, the design parameter is assumed to be distributed uniformly in its range. Afterwards, the d-th and $(d+1)$-th degree arbitrary polynomial chaos expansions are constructed repeatedly, whose input parameters contain uncertain parameters and the design parameter considered. Then, the cumulative distribution functions (CDFs) of T_e can be numerically calculated by combining LHS and the d-th degree and $(d+1)$-th degree arbitrary chaos expansions. If the difference between two CDFs is smaller than 5%, the $(d+1)$-th degree expansion of T_e can be used as the surrogate model. If the difference is significant, the $(d+2)$-th degree expansion should be reconstructed, and the above process is repeated until two CDF curves are similar.

Here, the coefficient of variation of the root-mean-square deviation, CV (RMSD) is adopted to quantify the difference between two CDF curves of T_e. The larger the value of CV(RMSD) is, the more significant the difference between two CDF curves is, and CV(RMSD) can be expressed as follows.

$$\text{CV(RMSD)} = \frac{\sqrt{\dfrac{\sum_{i=1}^{m}(T_{e,i}' - T_{e,i}'')^2}{m}}}{\dfrac{\sum_{i=1}^{m} T_{e,i}''}{m}} \qquad\qquad (6)$$

Where m is the number of points constituting the CDF of evacuation time; $T'_{e,i}$ is the $(i/m*100)$-th percentile point of one CDF curve of T_e; $T''_{e,i}$ is the $(i/m*100)$-th percentile point of another CDF curve of T_e based on the arbitrary polynomial chaos expansion of one degree higher than that used to obtain $T'_{e,i}$.

Post–Processing

Once the suitable degree for the surrogate model of T_e is determined in the range of a certain design parameter, the surrogate model of T_e associated with uncertain input parameters can be constructed repeatedly at different values of the design parameter .

For a certain value of the design parameter, the surrogate model of T_e with a suitable degree d' can be given in Eq. 7.

$$T_e(x'_1, x'_2, ..., x'_n) \approx \alpha_0 + \sum_{i1=1}^{n} \alpha_{i1}\phi^1(x'_{i1}) + \sum_{i1=1}^{n} \alpha_{i1,i1}\phi^2(x'_{i1}) + \sum_{1 \leq i1 < i2 \leq n} \alpha_{i1,i2}\phi^1(x'_{i1})\phi^1(x'_{i2}) + \cdots +$$

$$(\sum_{i1=1}^{n} \alpha_{i1,i1,\dots,i1}\phi^{d'}(x'_{i1}) + \sum_{1 \leq i1 < i2 \leq n} \alpha_{i1,i2,\cdots,i2}\phi^{(d'-1)}(x'_{i1})\phi^1(x'_{i2}) + \cdots)$$

(7)

Furthermore, due to the orthogonal and normalized properties of ϕ , the mean and variance of T_e, $E(T_e)$ and $var(T_e)$ can be analytically calculated as follows.

$$E(T_e) = \alpha_0$$

$$var(T_e) = \sum_{i1=1}^{n}(\alpha_{i1})^2 + \sum_{i1=1}^{n}(\alpha_{i1,i1})^2 + \sum_{1 \leq i1 < i2 \leq n}(\alpha_{i1,i2})^2 + \cdots + (\sum_{i1=1}^{n}(\underbrace{\alpha_{i1,i1,\dots,i1}}_{d'})^2 + \sum_{1 \leq i1 < i2 \leq n}(\underbrace{\alpha_{i1,i1,\cdots,i1,i2}}_{(d'-1)})^2 + \cdots)$$

(8)

Based on the definition of Sobol sensitivity indices [21], the main and total sensitivity indices of T_e for uncertain parameter x_i , S_{x_i} and $S^T_{x_i}$ can also be expressed as follows.

$$S_{x_i} = \frac{var(E(T_e|x_i))}{var(T_e)}$$

$$S^T_{x_i} = \frac{var(T_e) - var(E(T_e|\mathbf{x}_{-i}))}{var(T_e)} = \frac{E(var(T_e|\mathbf{x}_{-i}))}{var(T_e)}$$

(9)

Where $E(T_e|x_i)$ is the mean value (expected value) of T_e given the values of x_i , whose variance $var(E(T_e|x_i))$ is equal to the variance of $E(T_e|x'_i)$ due to the linear transformation of x_i ; $E(T_e|\mathbf{x}_{-i})$ is the mean value (expected value) of T_e given the values of non-x_i ; $var(T_e|\mathbf{x}_{-i})$ is the variance of T_e given the values of non-x_i, whose mean value (expected value) is equal to the mean value of $var(T_e|\mathbf{x}'_{-i})$ due to the linear transformation of x_i .

Since ϕ is normalized orthogonal basis, the variance of $E(T_e|x'_i)$ and the mean value of $var(T_e|\mathbf{x}'_{-i})$, $var(E(T_e|x'_i))$ and $E(var(T_e|\mathbf{x}'_{-i}))$ can be analytically computed, as shown in Eq.(10) and Eq.(11).

$$\mathrm{var}(E(T_e|x_i')) = \sum_{i1=i}(a_{i1})^2 + \sum_{i1=i}(a_{i1,i1})^2 + \ldots + \sum_{i1=i}\left(a_{\underbrace{i1,i1,\cdots,i1}_{d'}}\right)^2 \tag{10}$$

$$E\left(\mathrm{var}(T_e|\mathbf{x}_{-i}')\right) = \sum_{i1=i}(a_{i1})^2 + \sum_{i1=i}(a_{i1,i1})^2 + \sum_{1\le i1<i2=i\le n}(a_{i1,i2})^2 + \sum_{1\le i1=i<i2\le n}(a_{i1,i2})^2 + \cdots$$
$$+ \left(\sum_{i1=i}\left(a_{\underbrace{i1,i1,\ldots,i1}_{d'}}\right)^2 + \sum_{1\le i1<i2=i\le n}\left(a_{\underbrace{i1,\ldots,i1,i2}_{(d'-1)}}\right)^2 + \sum_{1\le i1=i<i2\le n}\left(a_{\underbrace{i1,\ldots,i1,i2}_{(d'-1)}}\right)^2 + \cdots\right) \tag{11}$$

Moreover, the CDFs of T_e can be numerically generated by combining the surrogate models of T_e and LHS. For different values of the design parameter, T_e at any level of uncertainty of interest can be obtained from the corresponding CDF. Based on the description above, the effect of the design parameter on T_e under arbitrary uncertainty can be examined.

The application of the arbitrary polynomial chaos-based method is demonstrated in the following case study, which is hypothetical due to the difficulty in available experimental data and scenarios.

CASE STUDY AND ANALYSIS

Case Description

The case considered is a single-room fire compartment of civil buildings, which is enclosed by fire-resistant construction to prevent the spread of fire to adjacent rooms. In order to make the analyzed fire compartment representative, the fire compartment is set to be 2500 m^2, which is the maximum area of a fire compartment of civil buildings regulated by the Chinese code GB50016-2012 [22]. Meanwhile, the minimum number of exits is 2 for this considered fire compartment prescribed by GB50016-2012. Given a worse case, two exits with the same size, denoted by Exit A and Exit B, are located at the centre of the walls. Additionally, in order to decrease the computational cost of evacuation simulations, obstacles are not taken into consideration in this fire compartment. Meanwhile, given emergent evacuation, occupants will escape immediately while notified by fire alarm systems. Thus, the pre-movement times for the crowd are assumed to 0 s here. Moreover, there is a possibility that fire may occur around a exit. Here, it is assumed that Exit B is not available while fire occurs. The plan of this considered fire compartment is shown in Fig.2.
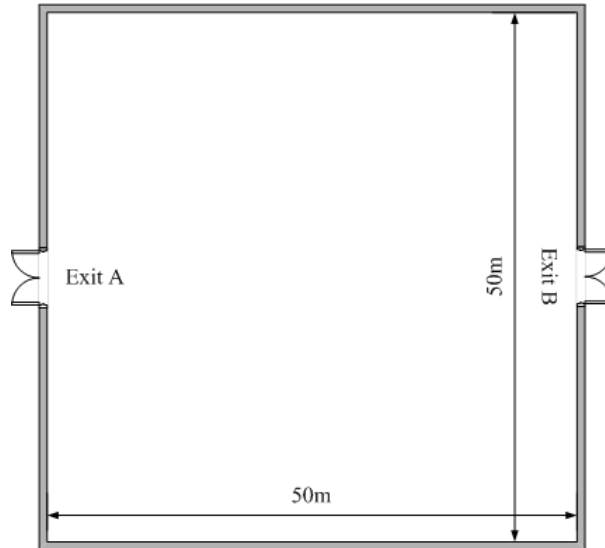

Fig. 2. The plan of the fire compartment considered

Due to the randomness of fire occurrence, it is difficult to accurately acquire the number and characteristics of evacuees. Thus, two uncertain parameters are considered here: occupant density and child-occupant load ratio. Compared with low density crowd evacuation, there is a higher risk in the high density situation. Here, the mean value of occupant density is assumed to be 1.0 person/m². Based on occupant type of "School Children" defined by SIMULEX [23], the mean value of the child-occupant load ratio is assumed to be 0.7 here. Moreover, occupant density and the child-occupant load ratio are assumed to follow normal distributions, which are truncated due to the bounded property. Meanwhile, the standard deviations of normal distributions for occupant density and the child-occupant load ratio are assumed to be 20% and 10% of their mean values. Table 1 gives the distributions of occupant density and child-occupant load ratio used in this case.

Table 1. Distribution of uncertain parameters considered

Uncertain parameters	Probability distribution functions
Occupant density, x_1/(persons/m²)	Truncated normal distribution: $f\left(x_1\right)=\dfrac{1}{\sqrt{2\pi}\sigma}\exp[-\dfrac{(x_1-u)^2}{2\sigma^2}]$ with $u=1, \sigma=0.2, 0.8\le x_1\le 1.2$
Child-occupant load ratio, x_2	Truncated normal distribution: $f\left(x_2\right)=\dfrac{1}{\sqrt{2\pi}\sigma}\exp[-\dfrac{(x_2-u)^2}{2\sigma^2}]$ with $u=0.7, \sigma=0.07, 0.56\le x_2\le 0.84$

In performance-based fire protection design of buildings, exit width are often paid great attention to by fire engineers or designers. When exit width is too small, people will not evacuate successfully. When exit width is too large, the available area of use will be reduced significantly. Here, the maximum and minimum exit widths are determined based on 0.5 m per 100 people [24] and 0.1 m for every 100 persons. According to the analysis above, the design range of exit width is shown in Table 2.

Table 2. Design parameter in the case

Design parameter	Range
Width for Exit A/m	2.5[a]-12.5[b]

[a]2.5 is calculated by the area of this fire compartment 2500 m² multiplied by occupant density 1.0 person/m² multiplied by 0.001 m/ person.
[b]12.5 is calculated by the area of this fire compartment 2500 m² multiplied by occupant density 1.0 person/m² multiplied by 0.005 m/ person that is chosen according to Ref. [24].

According to the proposed method, firstly, we should determine the suitable degree for the surrogate model of T_e associated with occupant density and child-occupant load ratio in the design range of exit width. Afterwards, surrogate models of T_e can be constructed repeatedly for different values of exit width. Finally, the analytical and numerical methods are adopted as post-processing to examine the effect of exit width on T_e under uncertainty caused by occupant density and child-occupant load ratio.

Results and Discussion

In order to determine the suitable degree for surrogate models of T_e in 2.5-12.5 m exit width, the 2nd, 3rd and 4th degree arbitrary polynomial chaos expansions of T_e are constructed based on 16, 41 and 86 evacuation time samples respectively, whose input parameters are occupant density, child-occupant load ratio and exit width. Then, LHS is applied to occupant density, child-occupant load ratio and exit width to obtain input samples. Afterwards, the CDFs of T_e can be generated by performing the 2nd, 3rd and 4th degree expansions on these input sample points, as shown in Fig. 3. For the 2nd and 3rd degree expansions, the value of CV(RMSD) between two CDF curves of T_e is 8.10%. However, for the 3rd and 4th degree expansions, it is 2.75% (smaller than 5%), which suggests that the 4th degree is suitable for the construction of surrogate models of T_e in the range of considered exit width. Furthermore, in order to verify the proposed method, the MCS can be applied to the FDS+Evac model. The sample size of the MCS that can be determined by statistical tolerance limits [1]. For the statistical tolerance limits (99.5%, 99.5%), which indicates that with

99.5% confidence level, 99.5% of the samples are in the tolerance limits, the required sample size is around 1000. Thus, the MCS of the FDS+Evac with 1000 sample size is performed in this case, and the corresponding CDF of T_e is shown in Fig. 3. For the 4th degree expansion with 86 samples and the MCS with 1000 samples, the value of CV(RMSD) between two CDF curves is 1.86%, smaller than 5%, which suggests that the 4th arbitrary polynomial chaos expansion of T_e is around the 99.5% confidence level. Thus in the range of 2.5-12.5m exit width, the appropriate degree for surrogate models of T_e is the 4th degree in this case study.
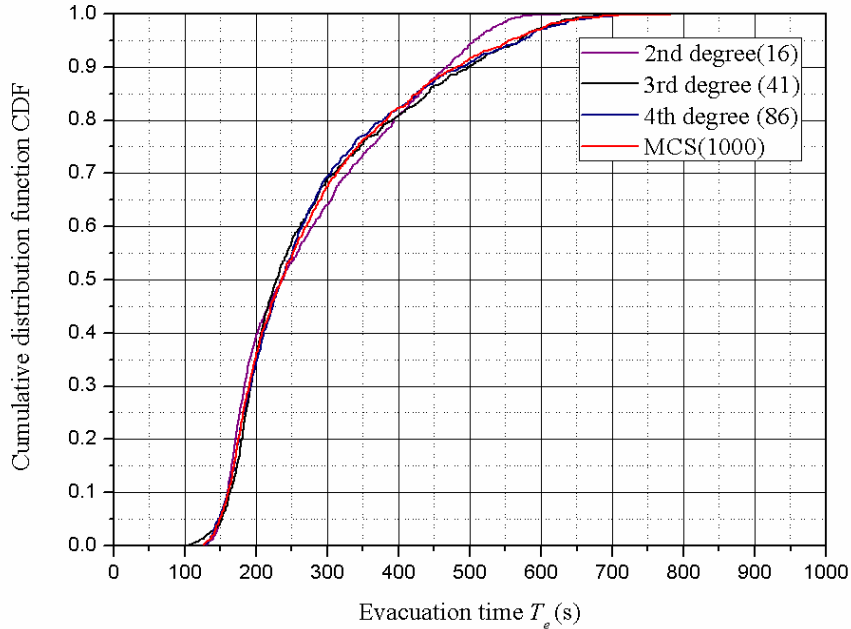

Fig. 3. Cumulative distribution functions (CDFs) of evacuation time T_e based on the 2nd, 3rd and 4th degree arbitrary polynomial chaos expansions and Monte Carlo simulation (MCS)

For a certain fixed value of exit width, 18 evacuation samples are required to construct one arbitrary polynomial chaos expansion of the 4th degree for T_e, whose input parameters are occupant density and the child-occupant load ratio in this case. To investigate the effect of exit width on evacuation time under uncertainty, the 4th degree expansions of T_e can be constructed repeatedly for different values of exit width. Afterwards, the corresponding mean value and standard deviation of T_e as well as Sobol total sensitivity indices can be calculated by analytical methods according to Eq.(8) and Eq.(9), as shown in Fig. 4.

From Fig. 4 (a) and (b), it can be seen that the mean value and standard deviation of T_e are reduced with the increase in exit width. For example, when exit width is 2.5 m, the mean value and standard deviation of T_e are 662.7 s and 71.0 s. However, when exit width is 12.5 m, they are 152.0 s and 15.4 s, which are much smaller than those for 2.5 m exit width. The explanation about this result is that while increasing exit width, the interaction between evacuees around exits will decrease and congestion and queuing phenomena around exits can be eased, which also suggests the effect of occupant density and child-occupant load ratio on T_e will decrease with the increase in exit width. Thus while increasing the exit width, T_e for a certain deterministic evacuation scenario will decrease and the difference in T_e for different scenarios will not be significant due to the decrease in the effect of occupant density and child-occupant load ratio on T_e. From the analysis above, it can be seen that the mean value and standard deviation of T_e for uncertain evacuation scenarios will decrease with the increase in exit width. Furthermore, when exit width is large enough, there are still strong interaction for high density crowd who are far away from exits during the movement process. From the analysis above, it can be seen that when exit width is large enough, the mean value and standard deviation of T_e for high density crowd will almost be unchanged with exit width. Furthermore, the Sobol total sensitivity indices shown in Fig. 4(c) are used to quantify the contributions of occupant density and child-occupant load ratio to the uncertainty of T_e. From Fig. 4(c), it can be seen that the contributions of

occupant density and child-occupant load ratio to T_e are little affected by exit width. Moreover, compared with child-occupant load ratio, the contribution of occupant density to T_e uncertainty is much larger. Thus, for the high density crowd evacuation, the child-occupant load ratio can be taken as the base value, whose uncertainty can be ignored.
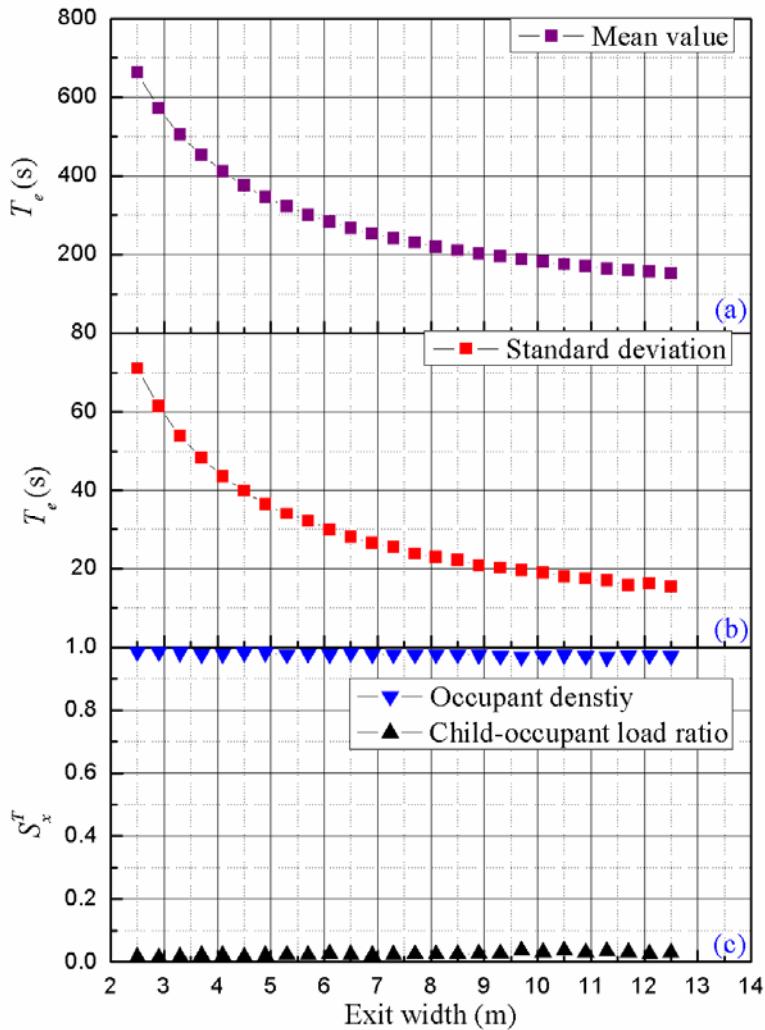

Fig. 4. Mean value (a), and standard deviation (b) of evacuation time T_e as well as Sobol total sensitivity indices S_x^T (c) versus exit width

Exit width and evacuation time uncertainty are key considerations in performance-based fire safety design. In order to obtain an optimal exit width under uncertainty, the effect of exit width on the distribution of T_e need to be investigated. Considering evacuation time at the level of uncertainty of interest, Fig. 4 gives the base case, 80th percentile, 90th percentile and 99.9[th] percentile evacuation times. The base case evacuation time is calculated from the 4th degree expansions of T_e constructed repeatedly for different values of exit width, whose uncertain parameters are taken as deterministic average values. The limitations of the RSET/ASET approach are widely known [25]. ASET may vary from 60-90 s in ultra-fast fire to 6-9 min in moderately fast growing fire [25]. The proper ASET value shall be the object of study considering the characteristics of each building, as a function of the inflammable material amount, type and distribution. In this case study, the exit width prescribed by the Chinese code GB50016-2012 (around 16.2 m wide for shopping malls) leads to a RSET around 120 s, corresponding to the ultra-fast growth rate fire. This exit may be oversized if there is no enough combustible material in this building. For a certain acceptable ASET, the optimal exit width at a certain reliability level can be determined from Fig. 5. For example,

when the acceptable ASET is 5 min, the optimal exit width at the reliability level of 99.9% should be selected around 7 m, as shown in Fig.5. Furthermore, for a certain exit width, T_e at a certain reliability level can also be obtained from Fig. 5. For example, when exit width is around 10 m, T_e at the reliability probability of 99.9% is around 225 s. If the corresponding ASET is smaller than 225 s, some measures should be taken, such as improving the detection and alarm system, installing the fire extinction and smoke control system.
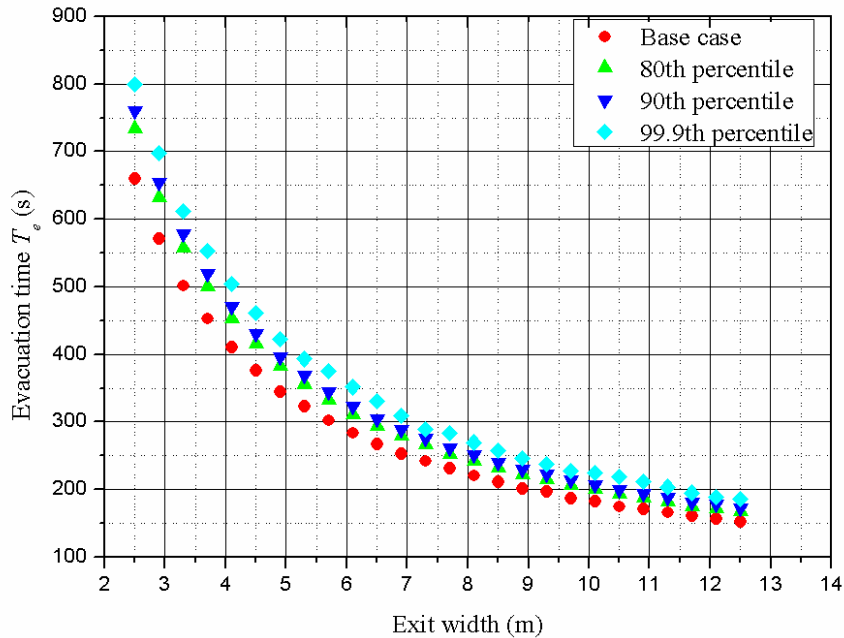

Fig. 5. Evacuation time T_e under uncertainty versus exit width

Due to the significant computational cost for the MCS of complex computer evacuation models, safety factors are generally adapted to deal with the uncertainty of evacuation time in fire protection design of buildings. However, the uncertainty of evacuation time is affected by many factors such as occupancy type, exit width, and the characteristics of evacuees, which makes safety factors random. Thus, the effect of exit width on uncertainty factor [26], which is defined as the ratio of evacuation time under uncertainty to the base case evacuation time, is investigated, as shown in Fig. 6. For a specified exit width, the optimal safety factor at a certain acceptable reliability level can be determined from Fig. 6. For example, when exit width is 8.5 m, the optimal safety factor at the reliability level of 99.9% is 1.22. In addition, for a specified exit width, the reliability probability of a certain selected safety factor can also be determined from Fig. 6. E.g. when exit width is 4.5 m, the reliability probability for the safety factor of 1.15 is 90%. Furthermore, from Fig. 6 it can also be seen that when exit width is designed in the range of 0.1 m and 0.5 m per 100 persons, exit width has little effect on the uncertainty factor at a certain reliability level.
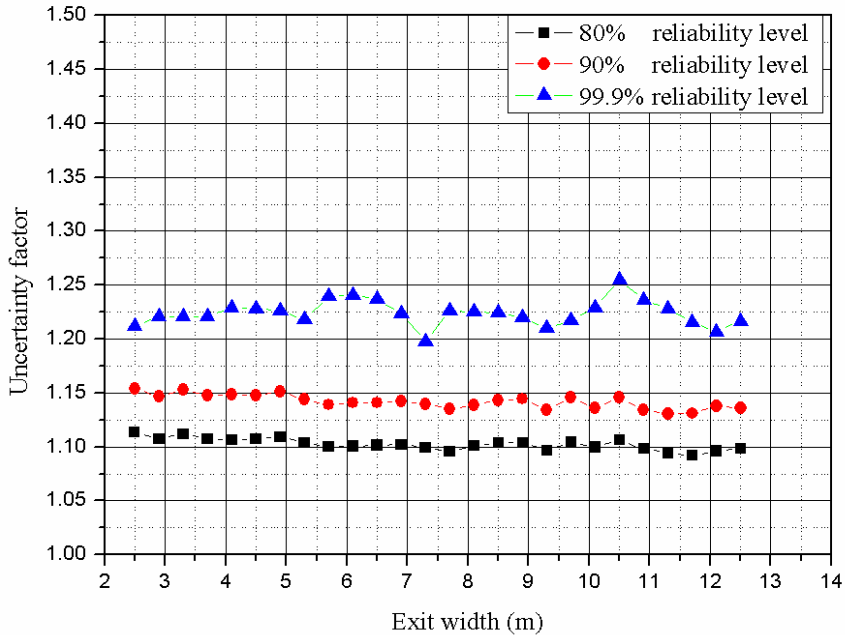
Fig. 6. Uncertainty factor versus exit width

CONCLUSIONS AND FUTURE WORK

In order to conduct a cost-effective performance-based fire protection design of buildings with the reduced computational cost, an arbitrary polynomial chaos-based method is presented to study the effect of design parameters on evacuation time under arbitrary uncertainty in this work. Based a case study of a single-room fire compartment, some conclusions can be summarized as follows.

The proposed method was demonstrated by the issue of the effect of exit width on evacuation time uncertainty resulting from occupant density and child-occupant load ratio in a square-shaped fire compartment. The results revealed that the proposed method can be used to investigate the impacts of design parameters on evacuation time under arbitrary uncertainty, whose computational cost is reduced significantly compared with the conventional MCS. In addition, the mean value and standard deviation of evacuation time as well as Sobol sensitivity indices can be calculated analytically on the basis of the arbitrary polynomial chaos coefficients, which do not need extra evacuation simulations.

In this case, it can be seen that evacuation time under uncertainty is significantly affected by exit width designed in the range of 0.1m and 0.5m per 100 persons. When exit width is small, the uncertainty of evacuation time is more significant. In addition, in this considered range of exit width, the contributions of occupant density and child-occupant load ratio to the uncertainty of evacuation time are not severely affected by exit width. Meanwhile, safety factors at a certain acceptable reliability level and the reliability level of a certain selected safety factor are almost independent of exit width. This method can also be used to optimise other design parameters in performance-based fire protection design, such as exit position.

However, there are still some limitations in this proposed method. First of all, we construct surrogate models of evacuation time based on the simulations of complex evacuation models, which makes the accuracy of the proposed method depend to a considerable degree on the selected evacuation models. Thus, it should be prudent to choose the evacuation model to construct the accurate enough surrogate model of evacuation time.

Secondly, this proposed method is applied to a hypothetical case, which has not be validated due to the difficulty of the available experimental data. In the future, we will focus on designing the realistic evacuation experiment to obtain the valuable evacuation data.

Thirdly, for the case of multiple uncertain parameters, it is computationally expensive to adopt the arbitrary polynomial chaos–based method to investigate the effect of the design parameter on evacuation time under

uncertainty due to the significant increase in the number of evacuation simulation runs with the number of uncertain parameters.

Finally, the proposed method only can be used to deal with the independent or linear correlation uncertain parameters. And the uncertainty of the output resulting from nonlinear correlation uncertain input parameters need to be addressed in our future work.

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