Uncertainties due to the use of surface averaged wind pressure coefficients

D. Cóstola, B. Blocken, J.L.M. Hensen

Building Physics and Systems, Eindhoven University of Technology, the Netherlands

ABSTRACT

A common practice, adopted by several building energy simulation (BES) tools, is the use of surface averaged wind pressure coefficients ($C_p$) instead of local $C_p$ values with high resolution in space. The aim of this paper is to assess the uncertainty related to the use of surface averaged data, for the case of a cubic building with two openings. The focus is on wind-driven ventilation and infiltration, while buoyancy is not taken into account. The study is performed using published empirical data on pressure coefficients obtained from wind tunnel tests. The method developed to calculate the uncertainty is based on comparison of: the flow rate calculated using the averaged values ($\phi_{AV}$), and the one calculated using local values ($\phi_{LOC}$). The study considers a large number of combinations for the opening positions in the facade. For each pair of openings ($i$), the values of $\phi_{LOC,i}$ and $\phi_{AV,i}$ are calculated. Based on the ratio between $\phi_{LOC,i}$ and $\phi_{AV,i}$ the relative error ($r_i$) is calculated. The relative error is presented statistically, providing probability density graphs and upper and lower bounds for the confidence interval (CI) of 95%. For this CI, the conclusion is that $0.24 \phi_{AV} < \phi_{LOC} < 4.87 \phi_{AV}$.

1. INTRODUCTION

Ventilation and infiltration air flow rates are important variables in building energy simulation and thermal comfort studies. Wind-driven ventilation and infiltration are complex phenomena and the calculation procedures are often simplified, introducing uncertainty in the analysis. The simplifications involve, for example: the wind data, the wind pressure distribution over the building facades and the characteristics of openings and cracks.

In this work, our attention is focused on the wind pressure, which is usually represented by wind pressure coefficients ($C_p$).

$C_p$ data for a specific building can be obtained from custom wind tunnel experiments. However, the costs, time and know-how involved in these experiments make them rare in building envelope air flow studies.

When custom $C_p$ data is not available, one usual solution is the adoption of generic databases published in the literature (Liddament, 1986; ASHRAE, 2001). Analytical models (Swami & Chandra, 1988; Grosso, 1992) are another common $C_p$ data source. In most of the cases (Liddament, 1986; ASHRAE, 2001; Swami & Chandra, 1988), $C_p$ is presented using surface averaged data, so the variation of $C_p$ across the facade is neglected.

Surfaced averaged $C_p$ ($C_{p-AV}$) data can be found in several building energy simulation (BES) software, e.g. ESP-r (Clarke, 2001) and EnergyPlus (EnergyPlus; 2007). $C_{p-AV}$ is also reproduced in a large number of publications, e.g. (Allard, 1998). The popularity of $C_{p-AV}$ is based on the simplicity of the data sets and analytical equations, which makes their use straightforward. The drawback is the increment in the airflow simulation uncertainty.

The limitations presented by $C_{p-AV}$ were one of the main motivations for the development of more sophisticated analytical methods, like (Grosso, 1992), as “From experience we know that wall-averaged values of $C_p$ usually do not
match the accuracy required for air flow calculation models.” (Feustel et al, 2005).

Swami & Chandra (1988) indicate a different direction regarding the averaging process. Based on early studies, they state that errors due to the surface averaging would be acceptable for low-rise buildings.

Despite the controversy, \( C_{p-AV} \) is still widely used. It is a reason for concern, because \( C_p \) is identified as one of the major sources of uncertainty in BES applications (Wit, 2001). The next section provides some examples of how the \( C_p \) surface averaging process can affect the air flow calculation.

1.1. Uncertainty on \( C_{p-AV} \)

Based on wind tunnel results (Quan et al, 2007), Figure 1 shows the histogram of \( C_p \) data for 5 faces of the cubic model, where the wind is perpendicular to one of the faces (\( \theta = 0^\circ \)) and each facade has 100 tappings. \( C_p \) varies in a large range, from -1.5 to 0.8, while the distribution is far from homogeneous, presenting clear peaks and gaps.

Figure 2 presents the histogram of the same data after the \( C_p \) surface averaging process. In this case, the data is reduced to 4 discrete values distributed over a smaller range. The most frequent value is also different from Figure 1.

The reduction in the spectrum of \( C_p \) values due to averaging may lead to errors in the flow rate calculation, but the errors depend on the position of the openings. Figure 3 and Figure 4 exemplify these two opposite situations.

![Figure 1: \( C_{p-LOC} \) histogram for a cube, \( \theta = 0^\circ \).](image1)

![Figure 2: \( C_{p-AV} \) histogram for a cube, \( \theta = 0^\circ \).](image2)

![Figure 3: Case 1 - \( C_{p-LOC} \) and \( C_{p-AV} \) have the same value.](image3)

Figure 3 presents a cube with a pair of identical openings at the position, say \( i=1 \). Figure 3 also provides the distribution of \( C_p \) over two surfaces of the cube (\( C_{p-LOC} \)), as well as the averaged values (\( C_{p-AV} \)). For these specific opening positions, the values of \( C_{p-LOC} \) and \( C_{p-AV} \) are the same. So, there is no difference between the flow rate calculated using \( C_{p-AV} \) (\( \phi_{AV_1} \)) and the one calculated using \( C_{p-LOC} \) (\( \phi_{LOC_1} \)).

In this case, the ratio between \( \phi_{AV_1} \) and \( \phi_{LOC_1} \) is equal to 1 (Equation 1), and the relative error (\( r_1 \)) is 0 (Equation 2).

\[
\frac{\phi_{LOC_1}}{\phi_{AV_1}} = 1 \quad (1)
\]

\[
r_1 = \frac{\phi_{LOC_1} - 1}{\phi_{AV_1}} = 0 \quad (2)
\]
Figure 4 shows the same cube, but the openings are now placed in another position, say i=2. In this case, $C_p_{-LOC}$ is quite different from $C_p_{-AV}$. So, $\phi_{LOC_2}$ will be higher than $\phi_{AV_2}$, because the real pressure difference is bigger than the surface averaged one, as presented in Equation 3. In this case, $r_2$ is equal to 0.5 (Equation 4), which means that the $\phi_{AV}$ is underestimated by 50%.

$$\frac{\phi_{LOC_2}}{\phi_{AV_2}} = 1.5$$  \hspace{2cm} (3)

$$r_2 = \frac{\phi_{LOC_2}}{\phi_{AV_2}} - 1 = 0.5$$ \hspace{2cm} (4)

2. MATERIAL AND METHODS

2.1. Experimental data

The “Tokyo Polytechnic University (TPU) aerodynamic database for low-rise buildings” (Quan et al., 2007) provided the experimental wind tunnel data used in this research. $C_p$ on each face of the cubic model was measured at 100 points, arranged in a regular array of 10 by 10 points.

Data are provided for 10 wind directions, from 0° to 45°, with intervals of 5°.

2.2. Relative error calculation

The relative error ($r$) for a specific set of openings (i) is defined as:

$$r_i = \frac{\phi_{LOC_{-i}}}{\phi_{AV_{-i}}} - 1$$ \hspace{2cm} (5)

For the particular case of two identical openings, with same area ($A$) and discharge coefficient ($C_z$), the flow rates ($\phi$) can be calculated according to Equation 6, where $U_{ref}$ is the reference wind speed at the building height.

$$\phi_{LOC_{-i}} = \frac{U_{ref} \cdot A \cdot C_z \cdot (\Delta C_{p-LOC_{-i}})^{0.5}}{\phi_{AV_{-i}} \cdot U_{ref} \cdot A \cdot C_z \cdot (\Delta C_{p-AV_{-i}})^{0.5}}$$ \hspace{2cm} (6)

From Equation 6 it is clear that $r$ does not depend on the reference wind speed ($U_{ref}$) and the opening characteristics ($A$ and $C_z$). Therefore, Equation 7 is used to calculate $r$ in this work.

$$r_i = \frac{\sqrt{\Delta C_{p-LOC_{-i}}}}{\sqrt{\Delta C_{p-AV_{-i}}}} - 1$$ \hspace{2cm} (7)

According to Equation 7, $\Delta C_{p-AV}$ must be different from 0. Due to this fact, Equation 7 is not suited to study the uncertainty when both openings are positioned in the same facade, where $\Delta C_{p-AV}$ is equal to zero. The same applies for cases where two facades have the same $C_p_{-AV}$, e.g. symmetric facades regarding the wind direction. The same $C_p_{-AV}$ value is also found, for some wind directions, in one leeward surface and in the roof. All those cases are not considered in this paper.

1.2. Objectives

As demonstrated in the previous section, the impact of the averaging process depends on the opening positions. Users of surface averaged $C_p$ data do not know the value of $r$ for the specific opening positions in their projects, but they can benefit from information about probable $r$ values when $C_p_{-AV}$ is used. This paper intends to calculate the range of $r$ values, for the confidence interval of 95%. In other words, it quantifies the uncertainty in the calculated flow rate when $C_p_{-AV}$ is used.

In order to achieve this goal, the particular case of a cubic building with two identical openings is adopted. The openings are positioned in different facades, so single sided ventilation and infiltration is not considered.
As demonstrated in section 1.1, the value of \( r \) depends on the position of the pair of openings. Hence, the calculation of the value of \( r \) must be performed for a representative number of opening pairs. For the cases where two or more faces do not have the same \( C_{p-\text{AV}} \) value, the relative error was calculated for a total of 100,000 opening pairs. In other cases, like \( \theta = 0^\circ, 10^\circ \) and \( 30^\circ \), 90,000 pairs were considered, while 80,000 pairs were used for \( \theta = 45^\circ \). Those results were treated statistically and are presented in the following section.

3. RESULTS

Figure 5 presents the probability density smoothed graph based on 100,000 values of \( r \), for \( \theta = 5^\circ \). As expected, the most probable errors are around zero. In these cases, the use of surface averaged values does not lead to major errors in the flow rate calculation.

![Figure 5: r probability density, \( \theta = 5^\circ \).](image)

Despite the expected peak around \( r = 0 \), the upper and lower tails show a large probability of high relative error in both directions. Figure 5 shows the limits for the confidence interval (CI) of 95%. Considering the amount of opening pairs used to construct this graph, this CI discards 2,500 pairs, in each tail.

The lower bound for CI = 95% is -0.75. It means that \( \phi_{\text{AV}} \) would be overestimating the real flow rate (\( \phi_{\text{LOC}} \)), which would correspond to only 1/4 of the calculated \( \phi_{\text{AV}} \) value.

The upper bound is 3.70, so \( \phi_{\text{AV}} \) would be underestimating \( \phi_{\text{LOC}} \), which is in fact almost 4 times higher.

Figure 5 presents results for only one wind direction, \( \theta = 5^\circ \). In the following graphs, the upper and lower bounds for other directions are presented.

Figure 6 shows the upper bound values for all wind directions, considering CI = 95%. The relative error varies from 3.87 to 0.53.

![Figure 6: Upper bound values of \( r \), CI = 95%.](image)

Values in Figure 6 present a large variation, indicating that some wind directions are associated with higher uncertainties. From Equation 7, it is possible to conclude that high \( r \) values may be associated with low \( \Delta C_{p-\text{AV}} \). Figure 7 presents the lower \( \Delta C_{p-\text{AV}} \) for each wind direction, and the same trend of Figure 6 can be observed. Low \( \Delta C_{p-\text{AV}} \) values happen between leeward surfaces and the roof, so the windward surface is not associated with high \( r \) values in the upper bound.

![Figure 7: Lower \( \Delta C_{p-\text{AV}} \) values.](image)
The upper bound value is highly influenced by openings in the roof, which are not present in several buildings. In order to understand the influence of the roof in the results, the calculation of r was repeated considering only the vertical faces, for approximately 60,000 opening pairs for each direction. Figure 8 presents the upper bound values for CI = 95%. The graph confirms that the higher values are associated with the roof, but the occurrence of high r values still persists.

Concerning the lower bound, the values for all wind directions lie in a smaller range, as shown in Figure 9. The maximum relative error is -0.76 for \( \theta = 10^\circ \), and \( \phi_{AV} \) will be highly overestimated for all directions. For the lower bound values, the roof is clearly not relevant.

From Equation 7 it can be seen that the lower bound values are associated with low \( \Delta C_p-LOC \) values. It happens for leeward faces where \( \Delta C_p-AV \) is not zero, but the \( \Delta C_p-LOC \) assume values close to zero. Again, the windward facade is not important in the bound value definition.

4. DISCUSSION

In this section, some of the limitations of this study are addressed.

The number of openings is limited to two, due to the methodology adopted. The use of more openings makes the problem dependent of the wind speed, the area of the openings and their discharge coefficients. In this case, results are more difficult to obtain and mainly, to present. For cases with several openings, it seems wiser to perform the uncertainty analysis for the building under study rather than try to obtain general values like those presented here. Multi-zone problems lie in the same situation.

The method presented in this paper is also not suited for the uncertainty analysis of combined wind and stack effects. Once more, the use of traditional methods for uncertainty assessment, e.g. Monte Carlo, can be used to address more complex and realistic cases.

The grid resolution adopted in the research, 10 per 10 points in each facade, is arbitrary. The grid resolution certainly has importance for points near the edges, where extreme \( C_p \) values occur. However, this is not a common position for openings, so the grid should not significantly affect the uncertainty results presented here. Even though it is a valid assumption, future applications of this method would benefit from grid sensitivity analysis, in order to obtain grid independent results.

For openings with exponents different from 0.5, e.g. some crack models, the method can be easily applied. It is clear that the higher the exponent, the higher is the influence of \( C_p \) in the calculated flow rate.

Another aspect regarding the opening description is the assumption of identical openings. There are several demonstrations that \( C_z \) depends on the external flow, e.g. (Costola & Etheridge, 2007), i.e. identical openings perform differently depending on their relative orientation to the wind direction. BES tools do not consider this phenomenon, so the assumption adopted here is in the same level of the state of the art in BES tools.

Sheltered buildings may also be the object of the method presented here. Considering that higher relative errors are associated with leeward surfaces, i.e. in suction areas, the sheltered buildings may present a similar...
behaviour. This fact makes the extension of this research to sheltered buildings even more relevant.

The results show that higher uncertainty is related to openings at the leeward facades and the roof. It does not lead to the conclusion that openings in the windward facade have no uncertainty, regarding the surface averaging process. The objective of this paper was to provide general values, and future work may address the uncertainty related only with pairs of openings where one is placed in the windward facade. However, the wind direction changes in time, so inevitably openings will lie in leeward surfaces at some moment.

Finally, the upper and lower limits calculated in this research can not be directly generalized for every $C_p$ data source that adopted surface averaged values. Several simplifications are present in the formulation and in the use of $C_p$ databases (Liddament, 1986; ASHRAE, 2001) and simple analytical models (Swami & Chandra, 1988).

Table 1 brings a brief list of some those simplifications, and this paper only addresses the first one, so the overall uncertainty is certainly higher than the values presented here.

Table 1: Simplifications on generic $C_p$ data sources.

<table>
<thead>
<tr>
<th>Factor that affects $C_p$</th>
<th>Common simplifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point of interest at the building facade surface</td>
<td>Surface averaged data</td>
</tr>
<tr>
<td>Wind profile</td>
<td>Assumed profile parameters in the building site (e.g. $\alpha$, $z_0$, $z_d$)</td>
</tr>
<tr>
<td>Sheltering elements (e.g. buildings, trees)</td>
<td>Obstructions with generic shape (e.g regular array of boxes)</td>
</tr>
<tr>
<td>Building geometry and facade detailing</td>
<td>Generic data used for any building shape, and no facade details considered</td>
</tr>
<tr>
<td>Wind direction</td>
<td>Low angular resolution</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The calculated uncertainty for a cube with two openings was provided, and a straightforward method to quantify the uncertainty was developed. The method does not depend on the opening type or wind speed, and future research may apply it to other building shapes and sheltering conditions.

The uncertainty magnitude is high, but the judgment about the usability of this data depends on the problem under analysis and the chosen performance indicator.

Higher relative errors are related to pairs of openings in the leeward facades and in the roof. The results provide boundaries for future improvements in the $C_p$ studies, and new developments can be compared to the uncertainty of the current methods.

6. ACKNOWLEDGEMENTS

This research is funded by the “Institute for the Promotion of Innovation by Science and Technology in Flanders” (IWT-Vlaanderen) as part of the SBO-project IWT 050154 “Heat, Air and Moisture Performance Engineering: a whole building approach”. This financial contribution is highly appreciated.

7. REFERENCES


